Current groups for non-compact manifolds and their central extensions

Karl-Hermann Neeb

Technische Universität Darmstadt Schlossgartenstrasse 7
64289 Darmstadt, Germany
email: `neeb@mathematik.tu-darmstadt.de`

Dedicated to Karl Heinrich Hofmann on the occasion of his 70th birthday

Abstract. In this paper we study two types of groups of smooth maps from a non-compact manifold M into a Lie group K which may be infinite-dimensional: the group $C_c^\infty(M, K)$ of compactly supported maps and for a compact manifold M and a closed subset S the group $C^\infty(M, S; K)$ of those maps which vanish on S, together with all their derivatives. We study central extensions of these groups associated to Lie algebra cocycles of the form $\omega(\xi, \eta) = [\kappa(\xi, d\eta)]$, where $\kappa\colon \mathfrak{k} \times \mathfrak{k} \to Y$ is a symmetric invariant bilinear map on the Lie algebra $\mathfrak{k}$ of K and the values of ω lie in $\Omega^1(M; Y)/dC^\infty(M; Y)$. For such cocycles we show that a corresponding central Lie group extension exists if and only if this is the case for $M = \mathbb{S}^1$. If K is finite-dimensional semisimple, this implies the existence of a universal central Lie group extension of the identity component of the current groups.

2000 Mathematics Subject Classification: 22E65; 58D15, 57T20.

Introduction

If M is a compact manifold and K a Lie group (which may be infinite-dimensional), then the so called current groups $C^\infty(M; K)$, endowed with the group structure given by pointwise multiplication, are interesting infinite-dimensional Lie groups arising in many circumstances. If M is a non-compact manifold, the full group $C^\infty(M; K)$ seems to be far too large to carry a Lie group structure compatible with its natural group topology, so that it is natural to study subgroups of maps $f\colon M \to K$ that either vanish outside a compact subset or decay fast enough at infinity. In the present paper we investigate the following two types of current groups on a non-compact manifold M. The first class consists of the groups $C_c^\infty(M; K)$ of compactly supported smooth maps and the second class of the groups $C^\infty(M, S; K)$ of maps on a compact manifold M for which all partial derivatives vanish on the closed subset $S \subseteq M$. The groups