# Preface

This monograph is devoted to a natural class of boundary problems for the Hodge-Laplacian, acting on differential forms. This class includes the absolute and relative boundary conditions, used in the Hodge-style representation of absolute and relative cohomology classes of the underlying domain by harmonic forms.

Continuing the program in [86] aimed at understanding the solvability properties of such boundary problems under minimal geometric and analytic regularity assumptions, here we push further the analysis developed in [50] of a layer potential attack on elliptic boundary problems on a class of domains introduced by Semmes [112] and Kenig and Toro [66], which we call regular Semmes-Kenig-Toro (SKT) domains. We initiate the study of boundary value problems for differential forms in this class of domains. In addition to the absolute and relative boundary conditions mentioned earlier, we also treat the Hodge-Laplacian equipped with classical Dirichlet, Neumann, Transmission, Poincaré, and Robin boundary conditions in regular SKT domains, with data in $L^p$ spaces, for arbitrary $p \in (1, \infty)$.

In a broad perspective, our results may be regarded as a natural completion, of an optimal nature, of the work initiated by E. Fabes, M. Jodeit, and N. Rivière in [32], whose scope is extended here through the consideration of differential forms in place of scalar functions, the (variable-coefficient) Hodge-Laplacian in lieu of the (constant coefficient) Laplace operator, and regular SKT subdomains of Riemannian manifolds, with arbitrary topology, replacing $\mathscr{C}^1$ domains with connected compact boundaries in the flat Euclidean setting.

In stark contrast to the scalar case from [32], the structural richness of the higher degree case considered here allows for a much larger variety of natural boundary value problems for the Hodge-Laplacian, which we formulate and study systematically via potential theoretic methods.

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