Preface

One of the most beautiful results in classical complex analysis that has a great appeal to geometry is the solution by F. Klein and H. Poincaré of the uniformization problem for multivalent analytic functions and (consequently) Riemann surfaces. It states that each conformal structure on a Riemann surface is induced by one of the three classical geometries: Euclidean, spherical or hyperbolic (Lobachevskian), that is this structure is represented by a Riemannian metric on the surface which has a constant (zero, positive or negative) curvature.

According to Felix Klein's Erlangen program of 1872, geometry is the study of the properties of a space which are invariant under a group of transformations. A geometry in Klein's sense is thus a pair (X, G) where X is a manifold and G is a Lie group transitively acting on X. Due to the Klein–Poincaré geometrization theorem, the Euclidean, spherical and hyperbolic geometries are the most important ones in dimension two. However, they are all particular cases of the more general *conformal geometry*, that is the $(S^2, \text{Möb}(2))$-geometry, where $\text{Möb}(n)$ is the group of conformal (Möbius) transformations of the n-dimensional sphere S^n. This is not a Riemannian geometry. A conformal structure on a manifold M is the same as a conformal class of Riemannian metrics, each locally conformally equivalent to a flat metric. In dimension three, due to Thurston's geometrization, many 3-manifolds admit conformal structures, although relatively simple ones might not (among the eight possible 3-geometries, nontrivial closed solvable and nilpotent manifolds are examples of this). Generally, conformal geometries naturally appear at infinity for negatively curved Riemannian geometries. Moreover, due to M. Gromov's [5, 6] geometric approach to infinite groups, conformal geometry invents new fruitful methods in combinatorial group theory.

The main goal of our book is to present the first systematic study of conformal geometry of n-manifolds, as well as its Riemannian counterparts (in particular, hyperbolic geometry). A unifying theme is the discrete holonomy groups of the corresponding geometric structures, which also involves algebra and dynamics. However, we do not pay much attention to 2-dimensional geometries covered by many classical and recent books (see, for example, Casson–Bleiler [1], Beardon [4], Ford [1], Kra [3], Maskit [12]). Also, this book does not cover conformal geometries that appear at