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Higher dimensional polarized varieties with non-integral nefvalue

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Abstract. Let X be an n-dimensional normal projective variety with terminal, Gorenstein, $\mathbb{Q}$-factorial singularities. Let L be an ample line bundle on X. Let τ be the nefvalue of (X, L). Then we classify (X, L), describing the structure of the nefvalue morphism of (X, L), when τ satisfies $n-k < \tau < n-k+1$ and $n \geqslant 2k-3$, $k \geqslant 4$. In the smooth case, we discuss the case $n = 2k-4$, $k \geqslant 5$, as well.

Key words. Complex polarized n-fold, ample line bundle, nefvalue, nefvalue morphism, Gorenstein, terminal, $\mathbb{Q}$-factorial singularities, adjunction theory, special varieties.

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Introduction

Let X be an n-dimensional projective variety with terminal, Gorenstein, $\mathbb{Q}$-factorial singularities and let L be an ample line bundle on X. If the canonical bundle K_X is not nef, the Kawamata rationality theorem and the Kawamata–Shokurov basepoint free theorem imply that there is a fraction $\tau = u/v$, with u, v positive coprime integers, and a morphism $\phi : X \to W$ with connected fibers onto a normal projective variety W such that $vK_X + uL \approx \phi^* H$ for an ample line bundle H on W and $u \leqslant \max_{w \in W}\{\dim \phi^{-1}(w)\} + 1$. We call τ the *nefvalue* and ϕ the *nefvalue morphism* of (X, L) respectively.

Thus $\tau \leqslant n+1$ and by the Kobayashi–Ochiai theorem $\tau = n+1$ if and only if $(X, L) \cong (\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(1))$.

It is a natural question to classify polarized pairs (X, L) in terms of the numerical values of τ and the structure of the morphism ϕ. The range $n-3 \leqslant \tau < n+1$ has been extensively studied by several authors. We refer to [4, Chapter 7] for the case $n-3 < \tau < n+1$ with $n \geqslant 5$, to [7] for the $n = 4$ case, to [11], [12] for the case $\tau = n-3$, and to [1] for a refinement in a more general context when ϕ is birational with $\tau = n-1, n-2$. Recently, the case where τ is not integer satisfying the condition $n-4 < \tau < n-3$, with $n \geqslant 5$ (as well as the case when τ satisfies $n-3 < \tau < n-2$), has been studied in [13].