# On spectral theory for singular $S$-hermitian difference systems

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## 1. Introduction

Recently Billigheimer has discussed in [1] a singular boundary-value problem for a five-term recurrence relation. These problems are the discrete analogues of singular boundary-value problems for a fourth-order differential equation of Sturm-Liouville type. For the regular case the author had pointed out in [7], that the theory of $S$-hermitian boundary value problems, developed by F. W. Schäfke and the author in [4], [5] and [6], can also be applied in a natural way to problems with difference equations. Here we show, that the corresponding singular theory for $S$-hermitian differential systems has its analogue for $S$-hermitian difference systems.

The methods used in this note are the same as developed for real differential systems by the author in [8], [9], [10], [11]. The general complex case was studied by H. D. Niessen in [3]. It seems, that the results of these papers are still unknown, because later on several notes were published, where special differential systems were studied. For example the reader may consider the paper of Kim [2]. Therefore we will not refer to the notes mentioned above in general, but we demonstrate the methods in connection with difference systems.

In Chapter 2 we begin with the general assumptions and discuss some simple conclusions. In Chapter 3 we consider the corresponding inhomogeneous equation and we show, that it has always a solution in an appropriate Hilbert space under suitable conditions. This leads in Chapter 4 to the definition of a selfadjoint resolvent and we give a characterization of all selfadjoint resolvents by selfadjoint boundary conditions. Here we repeat some results of [10], which is unpublished. In Chapter 5 we deduce the spectral theory for singular difference systems from the spectral theorem for selfadjoint operators in Hilbert spaces, and the main result is an expansion theorem for arbitrary elements not only in the norm topology, but also with respect to pointwise convergence. In the last Chapter 6 we indicate, how the results of Billigheimer can be expanded to $(2r + 1)$-order recurrence relations by writing them as special systems, and we verify, that in the case $r = 2$ the assumptions of Billigheimer are just the general assumptions of Chapter 2.

It is clear, that many other results from the theory of singular $S$-hermitian differential systems are also valid for difference systems. Thus for real systems we have a partition of the solution space into two-dimensional subspaces, in which the alternative of Weyl is true. For the corresponding result with differential systems see [12].