Modular quadratic and Hermitian forms over Dedekind rings. I*)

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In this paper, we consider bilinear and quadratic forms over a Dedekind ring S, taking values in S-modules. In general, we assume that S has an involution, and that our forms take values in an S-module P which comes equipped with an involutary automorphism extending, in some sense, the involution on S. We have in mind a generalisation of the modular lattices of [8]. For this and other reasons we will insist that P be projective of rank 1.

One can develop an abstract theory of bilinear forms taking values in such modules, which is essentially identical to the standard theory in which P is isomorphic to S. We indicate briefly how to do this in § 2. We are principally concerned with the comparison between our so-called modular forms, and forms over the quotient field of S. Our main tool is a homomorphism of Grothendieck groups of forms induced by extending the base ring from S to its quotient field. The study of this map falls into two parts, namely the computations of its kernel and its image. Here, we give only the kernel computations, and treat the images in a subsequent paper.

Our general set-up includes, as a special case, the usual situation in which one studies nonsingular forms taking values in S itself. Consequently, our results imply much that is known already; see, for example [4], [7], [10]. In some areas, the gain in generality is insignificant, and nowhere are the generalisations at all surprising. The methods required, however, have to be rather different from those elsewhere employed in the standard case. This stems from the fact that we do not have a stability theorem of the type of Bak's theorem (see "Algebraic K-Theory and its Geometrical Applications", Springer Lecture Notes vol. 108), and our Witt groups do not have a ring structure. Consequently, we have developed an ad hoc approach, based ultimately on that of [4]. It should be observed, however, that Bak's result does extend to this situation.

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