Real closures of commutative rings. II

By *Manfred Knebusch* at Regensburg

This is the second and final part of a paper [11] dedicated to Helmut Hasse on his 75th birthday. The emphasis will be on local studies, and the central result is the main theorem 10. 12 at the end of § 10. This is a theorem about semi-local rings with involution, which in the special case of trivial involution tells us, that the signatures of the semi-local ring A correspond uniquely to the conjugacy classes of elements of order 2 in the Galois group $G(\bar{A}|A)$.

In § 9 we present some applications of our results about real closures to the structure theory of Witt rings, partially announced already in [12]. In particular we try to obtain some information about the subring $N(A)$ of a Witt ring $W(A)$ generated by the "natural forms" over A, i.e. the forms $\mathrm{Tr}_{B/A}(x\bar{y})$ with B running through the finite etale extensions of A. This section is not needed for the proof of the main theorem and thus may be skipped by readers interested only in this theorem.

In § 11 we discuss the perhaps easiest global situation of interest, namely real closures of affine curves over the field of real numbers. Our results support the hope that here a theorem completely analogous to Theorem 10. 12 holds true, cf. Question 11. 11.

The terminology and notations developed in part I of the paper [11] will be used throughout without further explanation.

§ 6. The involution of a real closure

We need—also for later sections—some more terminology. Let A be a π-ring with trivial involution.

Definitions 6. 1. i) We call A *strictly simply connected*, if A is simply connected in the category of rings without involution.

ii) Let A be connected. Then we call a covering $\varphi: A \to T$ of A a *strict universal covering*, if T has trivial involution and T is strictly simply connected.

In § 5 we introduced the notation $\bar{A}$ for the unique subring of $\tilde{A}$ containing A which is a strict universal covering of A.

We now consider an arbitrary real closed pair (R, ϱ). We denote by ϱ_0 the restriction of ϱ to the fixed ring R_0 of the involution J_R. We know from Prop. 3. 17 that (R_0, ϱ_0)