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Estimates for reduced binary forms

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§ 1. Introduction

Every binary form $F(X, Y) = a_0 X^r + a_1 X^{r-1} Y + \cdots + a_r Y^r$ of degree $r \geqq 2$ can be factored over some algebraically closed field as $\prod_{i=1}^{r} (\alpha_i X + \beta_i Y)$. The *discriminant* of F is defined by

$$D(F) = \prod_{1 \leqq i < j \leqq r} (\alpha_i \beta_j - \alpha_j \beta_i)^2 .$$

$D(F)$ is a homogeneous polynomial of degree $2r-2$ in $a_0, \ldots, a_r$. It is easy to show that for $\lambda \neq 0$ and for non-singular matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

$$(1.1) \qquad D(\lambda F) = \lambda^{2r-2} D(F), \quad D(F_A) = (\det A)^{r(r-1)} D(F),$$

where $F_A(X, Y) := F(aX + bY, cX + dY)$. From arguments of Lewis and Mahler [16] it follows that if F has complex coefficients then

$$(1.2) \qquad |D(F)| \leqq r^{2r-1} H(F)^{2r-2},$$

where $H(F) := \max(|a_0|, \ldots, |a_r|)$ is the height of F.

We now consider binary forms with coefficients in $\mathbb{Z}$. Two such binary forms F, G are said to be *equivalent* if $G = F_U$ for some matrix $U \in GL_2(\mathbb{Z})$ (i.e. U has entries in $\mathbb{Z}$ and determinant ± 1). A binary form $F \in \mathbb{Z}(X, Y)$ is called *reduced* if $H(F) \leqq H(G)$ for every binary form G equivalent to F. By (1.1), two equivalent binary forms have the same discriminant. Hence binary forms of given discriminant can have arbitrarily large height. We are interested in estimates of the type

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