On the asymptotic normality of the number of absent s-chains*

M. I. TIKHOMIROVA

Abstract — The asymptotic behaviour of the number μ_0 of absent s-chains in a polynomial scheme is studied. We consider the case where the length of chains s is constant, $s \geq 2$, and the number of trials, the number of outcomes and their probabilities are related by the conditions determining an analogue of the central domain. Some asymptotic expressions for the mean and variance are given and the asymptotic normality is proved.

1. INTRODUCTION

In this paper some topics concerning the asymptotic behaviour of the number μ_0 of absent s-chains in a polynomial scheme are studied. We consider the case where the length of chains is constant, the number of trials n and the number of outcomes of the scheme N tend to infinity in such a way that

$$0 < \alpha_1 \leq \frac{n}{N^s} = \alpha \leq \alpha_2 < \infty,$$

and the probabilities of outcomes $p_1, \dots, p_N$ vary in such a way that

$$0 \leq N p_k \leq C, \qquad k = 1, 2, \dots, N. \tag{1}$$

These conditions are similar to the well-known conditions determining the 'central domain' of parameters of a polynomial scheme [1] and coincide with them for $s = 1$.

In the present paper our attention is focused on the case $s \geq 2$. In this case the statistic μ_0 is a decomposable statistic on the frequencies of outcomes of the Markov chain generated by s-chains. It seems to us that the time is appropriate to extend the investigations of such statistics since in the case of the proper polynomial scheme, i.e., in the case $s = 1$, the investigation of decomposable statistics has been principally finished.

As in the case $s = 1$, the analysis of the asymptotic behaviour in the 'central domain' is the most difficult one. Whereas for other relations between the parameters a certain progress was made rather quickly [2–4], the properties of decomposable statistics of s-chains in the 'central domain' remained unknown for a long time. Now the situation begins to change, as demonstrated by the paper [5] where it is proved that the conditions (formulated for the case μ_0)

$$n p_i^s \leq C < \infty; \qquad \lim_{n \to \infty} n^{-1} \mathrm{D} \mu_0 > 0$$

are sufficient for the asymptotic normality in a wide class of decomposable statistics of s-chains. In the case of 'central domain' (1) the problem reduces to finding a required