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The scheme complexity of discrete optimization*

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Abstract — We consider an approach to the problem of minimization of a linear form $(\tilde{a}, \tilde{x})$ over $\tilde{x} \in M \subset B_k^n$, where $\tilde{a} \in \mathbf{R}^n$ is an input vector, $M = \mathbf{N}_f = \{\tilde{x}\colon f(\tilde{x}) = 1\}$, $f(\tilde{x})$ is a characteristic function of k-valued logic. To estimate the complexity of search of the optimal point, we use the structure characteristic of the set $\mathbf{N}_f$, which is equal to the complexity of the scheme description of this set by means of functional elements in suitable bases. The complexity of descriptive schemes describing $\mathbf{N}_f$ and such that they are associated by a natural way with optimization schemes of the same complexity is also used.

It is shown that there are essential analogues with the problems of synthesis of computing schemes from functional elements including the applicability of analogues of methods of the theory of control system to discrete optimization. In many cases this approach gives polynomial algorithms, not always being the best. Sometimes we may obtain a polynomial algorithm by means of a preliminary processing of the input vector $\tilde{a}$.

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1. INTRODUCTION

In this paper we suggest an approach to the construction of algorithms and upper bounds of the complexity of discrete extremal problems, similar to the approach to the synthesis of control system [1–3], and close to the approach to the synthesis of schemes from functional elements. The problem of minimization of a linear form on a finite set of vectors with integer-valued components is considered. In the most general case the region of optimization M is described by means of a characteristic function of k-valued logic: $M = \mathbf{N}_f = \{\tilde{x}\colon f(\tilde{x}) = 1\}$.

Some statistic regularities and applicability of concrete heuristic methods are established (for example, the method of cascades [2]). We introduce the notion of a descriptive scheme with a vector input. Sets of vectors are constructed on the nodes of such scheme so that the set $\mathbf{N}_f$ is obtained on the output. If the basis B of operations is chosen in a suitable way then the scheme $S = S_f$ is associated naturally with a scheme algorithm S' of minimization of a linear form on $\mathbf{N}_f$ with complexity $L(S') = L(S)$, where $L(S)$ is the complexity of the scheme. In this case the complexity of the problem of discrete optimization is connected with the complexity of the description of the region of optimization, and in the cases where the last problem has a simple solution, it is also possible to obtain a simple algorithm of the solution of the optimization problem. It is shown that in some cases a preliminary processing of input information permits to change the structure of the region of optimization and to increase the effectiveness of scheme algorithms. The examples given in the paper show that scheme algorithms are effective in many problems in such a region as the optimal coding, where independent approaches to discrete extremal problems are developed (see, for example [4]).

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