Discrete Math. Appl., Vol. 3, No. 6, pp. 635–647 (1993)

Theorems on large deviations in the scheme of allocating identical particles into different cells*

A. N. TIMASHEV

Abstract — One-dimensional local and integral theorems on large deviations for the number of cells with a fixed number r of particles in the scheme of allocating n identical particles into N different cells, as $n, N \to \infty$ in the central domain, are obtained.

We consider the scheme of allocating n particles into N cells, where all allocations of particles into cells have the same probability $\binom{n+N-1}{n}^{-1}$. This scheme is mentioned in [1] as the Bose–Einstein statistics. Such a kind of allocation is considered in [2] in detail, where some limit theorems on distributions of random variables $\xi_r(n, N)$, $r = 0, 1, \ldots$, are formulated and proved. Here $\xi_r(n, N)$ is the number of cells containing exactly r particles. The classification of possible domains of variation of n, N and results of investigation of the limit distributions of the random variables $\xi_r(n, N)$ in these domains are presented in [3].

In this paper we obtain local and integral theorems on large deviations which give us the asymptotic estimates of the probabilities $\mathsf{P}\{\xi_r(n, N) = k\}$, $\mathsf{P}\{\xi_r(n, N) \geq k\}$ and $\mathsf{P}\{\xi_r(n, N) \leq k\}$ in the central domain [4] of variation of n, N as $n, N \to \infty$ in such a way that

$$0 < \alpha_0 \leq \alpha = \frac{n}{N} \leq \alpha_1 < \infty, \tag{1}$$

where α_0, α_1 are constants.

We assume that r is fixed and consider two cases: k is fixed and $k \to \infty$ in such a way that $0 < \beta_0 \leq \beta = k/N \leq \beta_1 < 1$, where β_0, β_1 are constants. Hereafter k is a non-negative integer.

The well-known (see [5, 6]) general results on the probabilities of large deviations for decomposable statistics allow us to obtain, under the conditions of the problem considered here, the asymptotic representations for the logarithm of the probability $\mathsf{P}\{\xi_r(n, N) \geq k\}$ without any estimation of the rate of convergence. The results given in this paper allow us to estimate the probabilities $\mathsf{P}\{\xi_r(n, N) \geq k\}$ and $\mathsf{P}\{\xi_r(n, N) \leq k\}$ themselves (and not only their logarithms) up to the factor $1 + O(1/N)$ in the mentioned above domains of variation of the parameters n, N, k. To obtain these results, we use the saddle point method (in one-dimensional and two-dimensional variants).

The distribution of the random variable $\xi_r(n, N)$ is defined by the relations

$$\mathsf{P}\{\xi_r(n, N) = k\} = \binom{N}{k}\binom{n+N-1}{n}^{-1} \frac{1}{2\pi i} \oint z^{kr-n-1} \left(\frac{1}{1-z} - z^r\right)^{N-k} dz \tag{2}$$

for $k = 0, 1, \ldots, N$ (see [2]).

*UDC 519.2. Originally published in *Diskretnaya Matematika* (1993) 5, No. 4, 29–42 (in Russian). Translated by A. V. Kolchin.