Forum Math. **12** (2000), 757–766

Forum Mathematicum

A formula for the multiplicity of the principal series in $L^2(\Gamma\backslash G)$

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(Communicated by Peter Sarnak)

Abstract. The purpose of this paper is to give a multiplicity formula of the principal series representation which appears in the decomposition of the regular representation on $\Gamma\backslash G$ as a certain limit of the integral representation in terms of the logarithmic derivative of the Selberg zeta function.

1991 Mathematics Subject Classification: 11F72, 11M41, 22E40.

0 Introduction

It is quite interesting to know the explicit formula of the multiplicity of each irreducible unitary representation in the decomposition of the regular representation of the non-compact Lie group G on a co-finite, or a co-compact quotient of a negatively curved Riemannian symmetric space by a discrete subgroup. It is, of course, well-known that the multiplicity of the unitary principal series is described as the order of a zero of the Selberg zeta function (see [G], [GW], [W]). In spite of this fact, if one wants to know the multiplicity as a very explicit quantity, and/or rather to find (uniform) bounds (if any) of multiplicities, it would be useful if there is another expression of the multiplicity. On the other hand, as to the case of the discrete series representations of G, it is also well-known that the multiplicity of a discrete series, say ω, which has a sufficiently regular parameter like an integrable discrete series has, is given by the volume of the corresponding locally symmetric space times the formal degree d_ω of the representation ω. (See e.g., [SW]).

The aim of the paper is therefore, to give a multiplicity formula of the principal series representation which appears in the decomposition of the regular representation on $\Gamma\backslash G$ as a certain limit of the integral representation. In other words, the

* Partially supported by Grant-in-Aid for Scientific Research (C) No. 10640009, the Ministry of Education, Science and Culture of Japan.

** Partially supported by Grant-in-Aid for Scientific Research (B) No. 11440010, the Ministry of Education, Science and Culture of Japan.