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Canonical bases for normal subgroups of finitely generated free groups with finite abelian factor groups

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Abstract. Free bases for normal subgroups of finitely generated free groups with finite abelian factor groups are constructed. These bases consist of powers of the generators and simple basic commutators in the generators of the free group. Applications concerning p-dimension subgroups (p a prime) and the construction of irreducible modules of linear groups are discussed.

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0. Introduction

Let $\mathscr{F}$ be a free group of rank d and let $\mathscr{G}$ be a subgroup of finite index n; it is well known that $\mathscr{G}$ is then a free group on $1 + n(d-1)$ generators. Furthermore, both *Nielsen's* and *Schreier's* methods (see for instance [Ha]) to show this result are constructive: they exhibit an explicit free basis for $\mathscr{G}$. However, we notice that in some applications (e.g. see Section 5) it may be useful to be able to impose some conditions on the form that the elements of such a basis will have, as words in the generators.

Our main result here (Theorem 1.3) is the construction of free bases C and D for a normal subgroup $\mathscr{N} \leq \mathscr{F}$, provided $\mathscr{F}/\mathscr{N}$ is finite abelian (for a basis of the commutator subgroup of $\mathscr{F}$ see [K]); these bases will consist of powers of the generators and simple basic commutators in the generators. The construction is completely elementary, since it only requires some standard commutator calculus. Using the basis D it is then easy to produce a further basis B which likewise could be constructed independently by a collecting process (see Section 4).

In Section 5 we remark that in principle we could construct a free basis for $\mathscr{N}$ provided $\mathscr{F}/\mathscr{N}$ is finite solvable, and we give an example in which such a construction is practical: namely, when $\mathscr{N}$ is one of the p-dimension subgroups of $\mathscr{F}$, for a prime p. Another application is the construction of modules for some groups. As an example we construct irreducible modules for 2-dimensional linear groups over a prime field.