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Existence theory for nonlinear hyperbolic systems in nonconservative form

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Abstract. This paper deals with the Cauchy problem for nonlinear strictly hyperbolic systems in nonconservative form. Weak solutions to these systems in the space of bounded functions of bounded variation are understood in the sense of Volpert [Vo] or, more generally, in the one proposed by Dal Maso-Le Floch-Murat [DLM]. Assuming that each characteristic field of the system is either genuinely nonlinear or linearly degenerate and the total variation of the initial data is sufficiently small, we prove the convergence of the random-choice method (introduced by Glimm in the context of systems of conservation laws). Our proof is based on the result of pointwise convergence due to the second author [Li3]. Our result proves the existence of entropy weak solutions of bounded variation for general systems in nonconservative form.

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1. Introduction

We consider the Cauchy problem for nonlinear hyperbolic systems in nonconservative form, i.e.:

$$\partial_t u + A(u)\,\partial_x u = 0, \quad u(t,x) \in \mathbb{R}^p,\ t > 0,\ x \in \mathbb{R}, \tag{1.1}$$

and

$$u(0,x) = u_0(x), \quad x \in \mathbb{R}. \tag{1.2}$$

In (1.1), A is a smooth matrix-valued function of the variable u possessing p real and distinct eigenvalues, denoted by $\lambda_1(u) < \lambda_2(u) < \ldots < \lambda_p(u)$ ($u \in \mathbb{R}^p$), and a corresponding basis of right eigenvectors $r_1(u), r_2(u), \ldots, r_p(u)$ ($u \in \mathbb{R}^p$). In (1.2), the initial data $u_0 : \mathbb{R} \to \mathbb{R}^p$ is assumed to belong to the space BV ($\mathbb{R}$, $\mathbb{R}^p$) of all functions of bounded total variation on $\mathbb{R}$. For the sake of simplicity, we also assume that each