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Products of simple isometries of given conjugacy types

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Abstract. Let (V, f) be a finite-dimensional regular orthogonal vector space over a field K and $\operatorname{char} K \neq 2$. Then every $\pi \in \mathrm{O}(V, f)$ is a product $\pi = \sigma_1 \cdot \ldots \cdot \sigma_k$ of at most $\dim V(\pi - 1) + 2$ symmetries. We find handsome conditions such that additionally the conjugacy classes can be imposed on $k - 1$ of these symmetries. Analogue theorems are proved for products of transvections in symplectic groups. The special case when K is a euclidean field is studied in greater detail.

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1. Introduction

1.1 Basic assumptions and preliminaries

Let K be a field of characteristic distinct from 2 and V a K-vector space. All vector spaces will be finite-dimensional. Let f: $V \times V \to K$ be a regular bilinear form such that the assigned orthogonality relation on V is symmetric; hence f is symmetric or alternating. Correspondingly, the group of isometries $G := \{\pi \in \mathrm{GL}(V) \mid f(a\pi, b\pi) = f(a, b) \text{ for all } a, b \in V\}$ is called an orthogonal group $\mathrm{O}(V, f)$ or a symplectic group $\mathrm{Sp}(V, f)$.

For a subspace W of V let $\operatorname{rad}(W, f) := \{w \in W \mid f(v, w) = 0 \text{ for every } v \in W\}$ denote the radical of W. Then f induces a regular bilinear form on $W/\operatorname{rad} W$. Call (V, f) isotropic if $f(v, v) = 0$ for some $v \in V \setminus O$.

For $\pi \in \operatorname{End}(V)$ let $B(\pi) := V(\pi - 1)$ be the path, $B^2(\pi) := V(\pi - 1)^2$ the second path, $F(\pi) := \operatorname{kernel}(\pi - 1)$ the fix and $N(\pi) := F(-\pi)$ the negative space of π. Clearly, $\dim B(\pi) + \dim F(\pi) = n$, $N(\pi) \subset B(\pi)$ and, for $\pi \in \mathrm{GL}(V)$, $B(\pi^{-1}) = B(\pi)$. Call π simple if $\dim B(\pi) = 1$. If $\pi \in G$ then $F(\pi)^\perp = B(\pi)$, in particular $\operatorname{rad} B(\pi) = B(\pi) \cap F(\pi)$ is the radical of $B(\pi)$.