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Arithmetic groups of gauge transformations

C. W. Stark

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Abstract. Linear groups π over many finitely generated $\mathbf{Z}$-algebras are realized as groups of functions over varieties. We show that this construction produces a large number of explicit $K(\pi, 1)$'s which are infinite dimensional double coset spaces.

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Introduction

The class of linear groups $\pi = G(A)$ over finitely generated $\mathbf{Z}$-algebras A includes the arithmetic groups as well as many which are less tractable, e.g. many groups of infinite virtual cohomological dimension. This paper shows that many linear groups $\pi = G(A)$ act as covering transformations or branched covering transformations on coset spaces of groups of maps, generalizing constructions familiar for arithmetic groups. These classifying spaces may also be viewed as quotients of spaces of maps into finite-dimensional symmetric spaces, and this description gives our spaces attractive geometric properties, such as curvatures which are computable for certain mapping functors by the methods of the appendix to [FG].

The groups of maps considered here may be viewed as the R-points of a linear algebraic group, where R is a subring of the coordinate ring of an affine variety, and this paper offers a characterization of a good class of rings R defining "arithmetic subgroups" of linear algebraic groups of functions. The rings R considered here are usually integral extensions of $\mathbf{Z}[X_1, \ldots, X_m, Y_1, Y_1^{-1}, \ldots, Y_n, Y_n^{-1}]$ $(m + n \geq 1)$ and the results and examples offered below indicate that they provide an appropriate setting for these constructions.

Although linear groups of maps over $\mathbf{Z}[X_1, \ldots, X_m, Y_1, Y_1^{-1}, \ldots, Y_n, Y_n^{-1}]$ and its integral extensions are usually discrete (with respect to the compact-open topology or any of the other common function space topologies), we find that discrete groups of maps from a space X to a Lie group G do not always act as covering transformations