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Division Rings and Group von Neumann Algebras

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Abstract. Let G be a discrete group, let $W(G)$ denote the group von Neumann algebra of G, and let $U(G)$ denote the set of closed densely defined linear operators affiliated to $W(G)$. If G is torsion free and has a normal free subgroup H such that G/H is elementary amenable, then we shall prove that there exists a division ring D such that $\mathbb{C}G \subseteq D \subseteq U(G)$. For G as above, this will establish the integrality of numbers arising from L^2-cohomology associated with G.

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1. Introduction

Let G be a group and let $L^2(G)$ denote the Hilbert space with Hilbert basis $\{g \mid g \in G\}$. Thus $L^2(G)$ consists of all formal sums $\sum_{g\in G} a_g g$ where $a_g \in \mathbb{C}$ and $\sum_{g\in G} |a_g|^2 < \infty$, and has inner product defined by

$$(\sum_{g\in G} a_g g, \sum_{h\in G} b_h h) = \sum_{g\in G} a_g \bar{b}_g$$

where $^-$ denotes complex conjugation. Let $\mathscr{U}$ denote the set of all closed densely defined linear operators [14, § 2.7] considered as acting on the left of $L^2(G)$, and let $\mathscr{L}$ denote the subset of $\mathscr{U}$ consisting of bounded operators. The adjoint θ^* of $\theta \in \mathscr{U}$ satisfies $(\theta u, v) = (u, \theta^* v)$ whenever $u, v \in L^2(G)$ and θu, $\theta^* v$ are defined. If $\alpha = \sum_{g\in G} a_g g \in \mathbb{C}G$ (so $a_g \in \mathbb{C}$ and $a_g = 0$ for all but finitely many g) and $\beta = \sum_{g\in G} b_g g \in L^2(G)$, then

$$\alpha\beta = \sum_{g,h\in G} a_g b_h gh = \sum_{g\in G} (\sum_{h\in G} a_{gh^{-1}} b_h) g \in L^2(G)$$

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