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A Note on the Fractal Nature of the Cellulose Fiber Surface

By Allan Johansson
Technical Research Centre of Finland, Biologinkuja 3–5, SF-02150 Espoo, Finland

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Summary

A simple model for the development of the cellulose fiber surface during beating is presented. The model involves subsequent self-similar transformations of the fiber ($A \Rightarrow A^* + m \cdot B$, $B \rightarrow B^* + m \cdot C$ etc) and seems to agree well with the experimental data of the surface irregularities of a fiber during beating.

Introduction

The measurement of the exposed surface of an individual fiber seems to be rather arbitrary and dependent on the degree of fitness of the contours, crevices, and fibrils taken into consideration by the method in use.

When the extent of the fiber surface is measured in terms of dry unit mass, the result is the specific surface area commonly called only specific surface. This is understood to picture the surface of a wet swollen fiber (in itself a state which is rather undefined). However, an even less controlled variable influencing such measurements is the quantity of small fragments "debris" present in the specimen. Such small fragments greatly influence the results of the surface measurement due to their high specific surface area. Many experimental methods exist for measuring the specific surface area (Clark 1978), generally relating surface area to some experimental quantities like density and water permeability through some phenomenological equations, often as a function of beating time.

In the following we will present a simple physical picture based on self similarity by which the qualitative development of such experimental quantities as specific area, specific surface and average fiber length seems to be well reproduced.

Simplistic Theoretical Model

The fiber is pictured to be a cylinder of radius r and length l. During beating this fiber undergoes successive self similar transformations in which a layer of the surface of the previous generation (the volume of which is equivalent to the total volume of the next generation) is peeled off, forming m new fibers scaled down by a factor k (per dimension). Although the model is overly simplified it is believed to contain some main features of the beating process.

As a layer is peeled of to form a new generation of branches on the fiber (or breaks of as new shorter fibers) it is reasonable to assume that in subsequent transformation the previous generations are "protected" from peeling by the lastly formed branches.

The new fragments can be freely suspended smaller fibers or fibrils attached to the previous generation as branches. Only the latter ones do in fact contribute to an increased contact surface between fibers with consequent increase in paper web strength. The peeled off fraction, after a few generations, eventually results in a fraction of debris.

This also explains why the development of the specific surface during beating does not correlate well with the development of burst or tensile strength, as only a diminishing part of the surface actually participates in fiber bonding, the rest representing the increasing specific surface of the debris.

A complete definition of fractals is still lacking (Feder 1988) but the following definition due to Mandelbrot (Feder 1988) illustrates the main points of fractals i.e. self similarity and the non-integer nature of the dimension of the measure.

1) A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension.

2) A fractal is a shape made of parts similar to the whole in some way.

The Hausdorff-Besicovitch dimension can be obtained by covering the set in the following way (Feder 1988):

A measure M_d of a set point in space is formed by a test function

$$h(\delta) = \gamma(d)\delta^d$$

$$M_d = \Sigma h(\delta) \qquad (1)$$

The Hausdorff-Besicovitch dimension D of a set S is the critical dimension for which the measure M_d changes from zero to infinity

$$M_d = \Sigma\gamma(d)\delta^d = \gamma(d)N(\delta)\delta^d \xrightarrow[\delta \to 0]{} \begin{cases} 0 & d > D \\ \infty & d < D \end{cases} \qquad (2)$$