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On an Improved Complex Tanh-function Method

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Abstract

An improved complex tanh-function method is introduced for constructing exact travelling wave solutions for both nonlinear partial differential equations and systems of nonlinear partial differential equations with complex phases and solutions.

Keywords: Improved tanh-function method, Complex tanh-function method, Hirota equation, Perturbed Wadati-Segur-Ablowitz equation, coupled Schrodinger-KdV equation.

1 Introduction

Recently, a number of methods for finding exact and numerical solutions of nonlinear partial differential equations were presented, such as the truncated Painleve expansion method[1], the Jacobi elliptic function expansion method[2-4], Adomian Pade approximation method[5] and F-expansion method [6-7] etc.

The most efficient and straightforward methods to construct exact solutions of partial differential equations are the extended tanh-function method [8, 9] and the complex tanh-function method [10].

The purpose of this paper is to improve the complex tanh-function method to find travelling wave solution for equations with complex phases [10, 11]. In Section 2, we introduce the improved complex tanh-function method. The improved complex tanh function method is then used to find the solutions of Hirota equation [12] in Section 3. In Section 4, this method is used to find the solution of the perturbed Wadati -Segur-Ablowitz equation [13]. The improved method is also applied to find travelling wave solution for the coupled Schrodinger -KdV equation [14, 15] in Section 5. Section 6 is devoted for conclusions.

2. The improved complex tanh-function method

It is useful to summarize the main steps for describing the improved complex tanh-function method:

1. Consider a general form of nonlinear partial differential equation (PDE)
$$N(u,u_t,u_x,u_{xx},\cdots)=0. \qquad (1)$$
2. To find the travelling wave solution of equation (1), we introduce the wave variable $\varsigma = kx+\omega t$ so that
$$u(x,t)=U(i\varsigma), \quad i=\sqrt{-1}, \qquad (2)$$

where k and ω are the wave number and the wave speed respectively. Thus, we use the following changes
$$\begin{aligned} &u_t=-i\omega U'(i\varsigma),\ u_{tt}=-\omega^2U'(i\varsigma),\\ &u_{xt}=-\omega k U''(i\varsigma),\ u_{tt}=-k^2U'(i\varsigma),\cdots, \end{aligned} \qquad (3)$$
Using (3) in (1), we obtain an ordinary differential equation (ODE) given by,
$$N(U,U',U'',\cdots)=0. \qquad (4)$$

3. If all the terms in (4) contain derivatives in ς, then by integrating this equation, and taking the constant of integration to be zero, we obtain a simple ODE.
4. Introduce the ansatz
$$u(x,t)=U(i\varsigma)=\sum_{s=0}^{n}a_sF^s(i\varsigma), \qquad (5)$$

where n is a positive integer that can be determined by balancing the linear term with the nonlinear term in equation (1) ; a_s, $s=1,2,\cdots,n$, are parameters to be determined and $F(i\varsigma)$ is a solution of the Riccati equation: