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Investigation of scattering and absorbing media by the methods of X-ray tomography

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Abstract — The aim of X-ray tomography is to determine the internal structure of a medium by analyzing how the radioactive particles pass through the medium. This problem is considered here as an inverse problem for the equation of radiation transport. We prove two uniqueness theorems for determining the attenuation coefficient provided that only the radiation intensity at the boundary of the medium is known. Formulae for the Radon transforms of the sought-for function are obtained and used to construct the algorithms for finding the attenuation coefficient. Computational examples for a test problem are presented. The results can be used in tomography for a medium with internal sources and quite arbitrary scattering of particles. The ideas of the previous work by D. S. Anikonov [1] are developed in this paper.

We consider an inverse problem for the transport equation which is to determine the main characteristic of a medium, namely, the radiation attenuation coefficient. Its specific feature is that the information provided for the attenuation coefficient is independent of the other characteristics of the medium. This is of major importance in tomography.

The basic concepts of the transport theory and exact formulations of inverse and direct problems are given in Section 1, as well as the method of determining the attenuation coefficient by choosing the external radiation which is discontinuous in an angular variable. The statements proved in Section 1 are not only of interest by themselves, but also form the basis of Section 2 where we solve the inverse problem by introducing a parameter. Therefore, the statements in Section 1 are more general than it is necessary for the former method.

In Section 2 we propose a method for determining the sought-for coefficient by constructing a singularity of the external source with respect to the parameter introduced. At the end of Section 2 we prove a theorem which is of practical interest in tomography for the determination of the boundaries of discontinuities in the materials composing the medium.

In Section 3 we consider two test problems for the methods and illustrate them by figures.

1. PROBLEM STATEMENT. SOLUTION FOR A DISCONTINUOUS EXTERNAL SOURCE

We consider the steady-state transport equation

$$(s, \operatorname{grad}_x \varphi(x,s)) + a(x)\,\varphi(x,s) = \frac{1}{4\pi}\int_\Omega b(x, s\cdot s')\,\varphi(x,s')\,\mathrm{d}s' + F(x,s). \tag{1.1}$$

The function $\varphi(x,s)$ is interpreted as the density of a particle flux at the point $x = (x_1, x_2, x_3)$ in the direction $s = (s_1, s_2, s_3)$, and $F(x,s)$ denotes the intensity of internal sources. The coefficients $a(x)$ and $b(x, s\cdot s')$ characterize the medium, they are called the

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