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Regularization of a class of nonlinear Volterra equations of a convolution type

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Abstract — A method of regularization of a class of nonlinear Volterra equations of a convolution type is analysed. The equations arise when solving inverse problems of determining the memory kernels in a heat flow.

1. INTRODUCTION

In [9] we reduce a class of inverse problems of identifying the memory kernels in a heat flow to integral equations of the type:

$$K_0 m(t) \equiv \int_0^t K[m](t-s)\,m(s)\,\mathrm{d}s = f(t), \qquad 0 \le t \le T \tag{1.1}$$

where the kernel K is a Volterra operator in the space $\{C[0,T] \to C[0,T]\}$. Under suitable conditions imposed on the data of the inverse problems there exists an integer $n \ge 1$ such that (1.1) becomes an equation of the second kind after we differentiate it n times. The integral operator K_0 is n-smoothing, i.e. $K_0 m \in C^n[0,T]$ if $m \in C[0,T]$. On the other hand, in applications the function f, which contains the observation data of the problem, is given approximately in $C[0,T]$. Since the problem is ill-posed in this space, we must apply regularization techniques.

In this paper we consider the general equation (1.1) with the above properties. We denote by f_δ the approximation of the function f and regularize (1.1) by the equation of the second kind:

$$L_{n,\varepsilon} m_{\varepsilon,\delta}(t) + \int_0^t K[m_{\varepsilon,\delta}](t-s)\,m_{\varepsilon,\delta}(s)\,\mathrm{d}s = f_\delta(t) + L_{n,\varepsilon} m_0, \qquad 0 \le t \le T. \tag{1.2}$$

Here

$$L_{n,\varepsilon} u(t) = \begin{cases} \varepsilon^n u(t) + \sum\limits_{i=1}^{n-1} \binom{n}{i} \varepsilon^{n-i} \int_0^t \frac{(t-s)^{i-1}}{(i-1)!}\, u(s)\,\mathrm{d}s, & n > 1 \\ \varepsilon u(t), & n = 1 \end{cases} \tag{1.3}$$

$\binom{n}{i} = \frac{n!}{i!(n-i)!}$, the parameter $\varepsilon > 0$, $m_0 = m(0)$, and

$$f_\delta \in C[0,T], \qquad \|f - f_\delta\|_{C[0,T]} \le \delta. \tag{1.4}$$

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