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On spectra of pairs of Poincaré-Steklov operators

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Abstract – A generalized spectral problem $\mathcal{P}_1 = \lambda\mathcal{P}_2$ is considered for two Poincaré–Steklov operators [3,4] corresponding to two adjacent domains in a plane. A sufficient condition for the problem's spectrum to be discrete is suggested. A continuous spectrum is known to impair the convergence of domain-decomposition iterative methods.

A Poincaré–Steklov operator is understood to be the operator that transforms the boundary values of a harmonic function to the values of the normal derivative of the function. It is shown that under certain conditions imposed on the domains the spectrum of this problem is discrete, it converges to $\lambda = 1$, and all the eigenvalues other than $\lambda = 1$ have finite multiplicities. This implies that the eigenvalues, except for a finite number of them, are contained in a small neighbourhood of unity. Moreover, it is shown that the spectrum of every problem of this kind is related to the spectrum of an integral operator with an analytic kernel, which can be sometimes calculated explicitly. It is also emphasized that the spectrum is invariant under conformal mapping of the two domains. The problem is investigated by the methods of the theory of complex-variable functions and the theory of one-dimensional singular equations.

1. INTRODUCTION

A generalized spectral problem with two Poincaré–Steklov ($\mathcal{P}$) operators

$$\mathcal{P}_1 u = \lambda \mathcal{P}_2 u \tag{1.1}$$

is encountered when justifying domain-decomposition iterative methods. Lebedev [3] put forward several hypotheses concerning the structure of the spectrum and the condition for the system of eigenfunctions (1.1) to be complete. In particular, in [3] the spectrum of the problem is assumed to be discrete, since a continuous spectrum impairs the convergence of iterative methods constructed. In the present paper the simplest problem for two adjacent plane domains is considered, with the Laplacian operator being the differential operator in both domains. Its geometric interpretation is as follows: one has to find functions, which are harmonic in either of the domains, vanish on the outside boundary of the domains, coincide on the common boundary, and have normal derivatives that differ by a factor of λ. Under certain restrictions on the domains (of practical importance is only the requirement for the angles adjacent to the common part of the boundary to be pairwise equal), it has been proved that the spectrum of (1.1) can be obtained by translating by unity the spectrum of a special integral operator with the analytic kernel depending on the domains. Thus, sufficient conditions for the spectrum to be discrete are obtained. There is an example of the problem in which the violation of the above condition for the angles to be pairwise equal brings into existence a continuous part in the spectrum. In this example the spectrum is calculated explicitly.

In this paper the spectral problem is treated in a space C^{μ} with the shift function dependent only on the above angles rather than in the space $W_2^{1/2}$, proper to the Poincaré–Steklov operator. The study of the operator $\mathcal{P}$ in its proper space calls for further investigation.