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Inverse problem for symmetric tridiagonal matrices. Calculation of the system of discrete orthogonal polynomials with arbitrary weight

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Abstract – Numerical methods for constructing symmetric tridiagonal matrices with prescribed distinct eigenvalues are studied. The first components of orthonormal eigenvectors or 'symmetry' conditions are additionally given. An analytic formula for the eigenvectors is derived. Using numerical examples we show that the Lanczos method with an additional complete 'forced' orthonormalization gives perfect results even for high-order matrices. Without the additional complete 'forced' orthonormalization, good results are obtained for low-order matrices only. An algorithm for calculating the system of discrete orthogonal polynomials with arbitrary weight is proposed.

1. INTRODUCTION

This work is devoted to numerical solution of an inverse 'symmetric' problem for symmetric tridiagonal matrices. The problem is encountered when reconstructing spectra in discrete quantum mechanics [7]. The wave motion is described in discrete quantum mechanics by the finite-difference Schrödinger equation [6]

$$-\frac{\psi(n+1) - 2\psi(n) + \psi(n-1)}{2\Delta^2} + V(n)\psi(n) = E\psi(n), \qquad n = 1,\ldots,N$$

$$\psi(0) = \psi(N+1) = 0.$$

When $V(n) = 0$, we have an eigenvalue problem for the matrix

$$D = \begin{bmatrix} 2 & -1 & 0 & \ldots & 0 & 0 \\ -1 & 2 & -1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 2 & -1 \\ 0 & 0 & 0 & \ldots & -1 & 2 \end{bmatrix}.$$

The eigenvalues of D are known to be

$$\lambda_j = 2 - 2\cos\left(\frac{j\pi}{N+1}\right), \qquad j = 1,\ldots,N$$

with the corresponding orthonormal basis of eigenvectors E_j defined by

$$E_j(i) = \left\{\sin\left(\frac{ij\pi}{N+1}\right)\Big/\sqrt{\frac{N+1}{2}}\right\}, \qquad i,j = 1,\ldots,N.$$