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Iterative grid methods for singularly perturbed elliptic equations degenerating into zero-order ones

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Abstract – A Dirichlet problem is considered on a rectangle for singularly perturbed linear and quasi-linear elliptic equations. When the perturbation parameter equals zero, elliptic equations degenerate into zero-order ones. Special iteration-free and iterative difference schemes which converge uniformly with respect to the parameter are constructed for boundary-value problems. Schwarz' method and the domain decomposition method are used to construct the schemes. Necessary and sufficient conditions are given for the solutions of the iterative difference schemes to converge uniformly with respect to the perturbation parameter as the number of iterates increases.

Various approaches to forming difference equations have been developed in order to construct grid approximations to problems of mathematical physics, in particular, to the equations that have smooth enough coefficients and solutions (see, for example, [9,10]). Methods of constructing difference schemes on the basis of alternating Schwarz' method and the domain decomposition method are fairly attractive, with them we can reduce the original problem to a sequence of subproblems on subdomains of a simpler form [5,19]. In addition, these methods allow parallel computation on multiprocessor computers [9].

The solutions of singularly perturbed boundary-value problems are of limited smoothness, and the accuracy of an approximate solution obtained with a classical difference scheme therefore essentially depends on the value of the parameter and may be comparable with the exact solution itself when the parameter is small [6]. Special approaches are used when developing difference schemes for such problems. In fitting methods the coefficients in the difference equations are chosen (fitted) so as to obtain a uniform accuracy of the approximate solution with respect to the perturbation parameter (see, for example, [1,3,6]). In methods of specially condensing grids the uniform approximation of the solutions to classical difference equations is obtained by rearranging the grid nodes (see, for example, [2]). The author has shown [14] that fitting methods are of limited usefulness. For instance, for problems with a parabolic boundary layer there is no fitting scheme that would converge uniformly with respect to the parameter. Some versions of Schwarz' method for specially condensing grids have been used when constructing the schemes for boundary-value problems in the domains with smooth boundaries [15,16].

In this paper on the basis of alternating Schwarz' method and the domain decomposition method, we devise methods for constructing iterative difference schemes of solving a Dirichlet problem for singularly perturbed linear and quasi-linear elliptic equations on a rectangle. When the parameter equals zero, differential equations degenerate into zero-order equations. As the parameter tends to zero, a boundary layer appears in the neighbourhood of the domain boundary. In the neighbourhood of smooth sections of the boundary the layer is regular and described by an ordinary differential equation, while in the neighbourhood of corner points we get a corner