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Asymptotic behaviour of solutions of nonhomogeneous linear systems of stochastic differential equations with constant coefficients

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Abstract—In this paper the limit behaviour of the solutions of the system

$$\mathrm{d}x(t) = (Ax(t) + \varphi(t))\,\mathrm{d}t + (Bx(t) + \psi(t))\,\mathrm{d}w(t)$$

is investigated for $t \to \infty$ if the system

$$\mathrm{d}y(t) = Ay(t)\,\mathrm{d}t + By(t)\,\mathrm{d}w(t)$$

is stable with probability 1.

Let us consider the system

$$\mathrm{d}x(t) = (Ax(t) + \varphi(t))\,\mathrm{d}t + (Bx(t) + \psi(t))\,\mathrm{d}w(t), \tag{1}$$

where A and B are $(n \times n)$-matrices,

$$\varphi(t) = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t)), \qquad \psi(t) = (\psi_1(t), \psi_2(t), \dots, \psi_n(t))$$

are vector-columns, $w(t)$ is the one-dimensional Wiener process, $w(0) = 0$. As it is known from [1], the solution of the system (1) may be represented in the form

$$x(t) = H_0^t x + H_0^t \int_0^t \left(H_0^s\right)^{-1}[\varphi(s) + B\psi(s)]\,\mathrm{d}s + H_0^t \int_0^t \left(H_0^s\right)^{-1}\psi(s)\,\mathrm{d}w(s),$$

where H_0^t is the matrix-solution of the system

$$\begin{aligned} \mathrm{d}H_0^t &= AH_0^t\,\mathrm{d}t + BH_0^t\,\mathrm{d}w(t), \\ H_0^0 &= E, \end{aligned}$$

E is the identity matrix.

Assuming that $\psi(t)$ is a smooth function and $\det B \neq 0$, we have

$$\begin{aligned} \mathrm{d}\left[\left(H_0^t\right)^{-1}B^{-1}\psi(t)\right] = {} & \left[\left(H_0^t\right)^{-1}(-A + B^2)B^{-1}\psi(t) + \left(H_0^t\right)^{-1}B^{-1}\frac{\mathrm{d}\psi(t)}{\mathrm{d}t}\right]\mathrm{d}t \\ & - \left(H_0^t\right)^{-1}\psi(t)\,\mathrm{d}w(t). \end{aligned}$$

Translated by the author