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Tauberian theorems for correlation functions and limit theorems for spherical averages of random fields

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Abstract—Tauberian and Abelian theorems for integral transforms of Hankel type are proved. The limit theorems for spherical averages of functionals of homogeneous isotropic Gaussian random fields are considered.

1. INTRODUCTION

Let $\mathbb{R}^n$ be an n-dimensional Euclidean space, $s(r) = \{x \in \mathbb{R}^n\colon \|x\| = r\}$ a sphere in $\mathbb{R}^n$, and $v_n(r) = \{x \in \mathbb{R}^n\colon \|x\| < r\}$ be a ball in $\mathbb{R}^n$. Let $\xi(x)$, $x \in \mathbb{R}^n$, be a real measurable square-mean continuous homogeneous isotropic Gaussian random field with $\mathbf{E}\xi(x) = 0$, $\mathbf{E}\xi^2(x) = 1$, and the correlation function $B_n(r) = B_n(\|x\|) = \mathbf{E}\xi(0)\xi(x)$.

It is known [1] that there exists a bounded nondecreasing function $\Phi(\lambda)$, $\lambda \geqslant 0$, such that

$$B_n(r) = 2^{(n-2)/2}\Gamma\left(\frac{n}{2}\right)\int_0^\infty J_{\frac{n-2}{2}}(\lambda r)(\lambda r)^{(2-n)/2}\Phi(\mathrm{d}\lambda), \tag{1}$$

where $J_\nu(z)$ is the νth-order Bessel function of the first kind and

$$\int_0^\infty \Phi(\mathrm{d}\lambda) = B_n(0) = 1.$$

It follows from the results of [1, 2] that

$$\begin{aligned} l_n(r) &= \mathbf{D}\left[\int_{s(r)} \xi(x)m(\mathrm{d}x)\right] \\ &= (2\pi)^n r^{2(n-1)} \int_0^\infty J^2_{\frac{n-2}{2}}(\lambda r)(\lambda r)^{2-n}\Phi(\mathrm{d}\lambda) \\ &= \frac{2^n \pi^{n-1}}{(n-2)!} r^{n-1} \int_0^{2r} z^{n-2}\left(1 - \left(\frac{z}{2r}\right)^2\right)^{(n-3)/2} B_n(z)\,\mathrm{d}z, \end{aligned} \tag{2}$$

where $m(\mathrm{d}x)$ is an element of Lebesgue measure on the sphere $s(r)$.

In the paper we study the relation between the asymptotic behaviour of the function $\Phi(\lambda)$ as $\lambda \to +0$ and that of the function $l_n(r)$ as $r \to \infty$.

We note that (1) may be treated as a Hankel type transform. Using the inversion formula [1] and the results of Bingham [3], we can obtain such a statement: $\Phi(\lambda)/\lambda^\alpha \sim$

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