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A problem of minimax smoothing for homogeneous isotropic on a sphere random fields

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Abstract—The problem of the least in a square-mean linear estimation for the transformation

$$A\xi = \sum_{j=0}^{\infty} \int_{S_n} a(j,x)\xi(-j,x)m_n(\mathrm{d}x)$$

of a homogeneous isotropic on a sphere S_n random field $\xi(j,x)$, $j \in \mathbb{N}$, $x \in S_n$, using observations of $\xi(j,x) + \eta(j,x)$ for $j \leqslant 0$, $x \in S_n$, where $\eta(j,x)$ is a homogeneous isotropic on a sphere S_n random field uncorrelated with $\xi(t,x)$, is considered. The least favourable spectral densities and the minimax (robust) spectral characteristics are determined for some classes of spectral densities.

1. Let S_n be the unit sphere in the n-dimensional Euclidean space, $m_n(\cdot)$ be the Lebesgue measure on S_n,

$$S_m^l(x), \qquad x \in S_n, \quad m = 0, 1, \ldots, \quad l = 1, 2, \ldots, h(m,n),$$

be the orthonormal spherical harmonics of degree m,

$$h(m,n) = (2m+n-2)(m+n-3)!((n-2)!m!)^{-1}$$

being the number of linearly independent spherical harmonics of degree m (for properties of spherical harmonics, see [1–3]). Let $\xi(j,x)$ be a continuous in a square-mean random field on $\mathbb{N} \times S_n$. We call the random field $\xi(j,x)$ homogeneous isotropic on a sphere if

$$\mathbf{E}\xi^2(j,x) < \infty, \qquad \mathbf{E}\xi(j,x) = 0$$

and

$$\mathbf{E}\xi(j,x)\xi(k,y) = B(j-k, \cos\langle x,y\rangle),$$

where $\cos\langle x,y\rangle = (x,y)$ is the "angular" distance between the points $x, y \in S_n$.

A homogeneous isotropic on a sphere random field has the form [4]

$$\xi(j,x) = \sum_{m=0}^{\infty} \sum_{l=1}^{h(m,n)} S_m^l(x)\xi_m^l(j), \tag{1}$$

$$\xi_m^l(j) = \int_{S_n} \xi(j,x)S_m^l(x)m_n(\mathrm{d}x),$$

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