

On measures of the excess between two concentric cylinders for homogeneous isotropic Gaussian random fields over certain level

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Abstract—The asymptotic distributions for the functionals

$$\{x \in v(r_2) \setminus v(r_1): \xi(x) > a(r_1)\}$$

and

$$\{x \in v(r_2) \setminus v(r_1): |\xi(x)| > a(r_1)\}$$

are studied, where $v(r_1)$ and $v(r_2)$ are two concentric balls in \mathbb{R}^n with radii r_1 and r_2 ($r_1 < r_2$), $|\cdot|$ is the Lebesgue measure, $a: \mathbb{R}^n \rightarrow \mathbb{R}^1$ is a non-random continuous radial function and $\xi(x)$ is a homogeneous isotropic Gaussian random field such that the integral of the correlation function diverges.

1. INTRODUCTION

In this article we consider limit distributions of functionals which are measures of random formations between two concentric cylinders, generated by the intersection of a random field with certain radial function. Our main attention is drawn to Gaussian random fields such that the integral of the correlation function diverges.

Problems involving sojourns of stationary Gaussian processes with a long range dependence have been studied by Berman [1] and Maejima [2].

2. MAIN RESULTS

Assume that $v(r_1)$ and $v(r_2)$ are two concentric balls in the n -dimensional Euclidean space \mathbb{R}^n with radii r_1 and r_2 ($r_1 < r_2$), i.e.

$$v(r) = \{x \in \mathbb{R}^n: |x| \leq r\},$$

and

$$|v(r)| = r^n c_1(n), \quad c_1(n) = \pi^n / \Gamma(n/2 + 1),$$

is the volume of the ball $v(r)$.

A. Let

$$\xi(x) = \xi(w, x): \Omega \times \mathbb{R}^n \longrightarrow \mathbb{R}^1$$

be a measurable square-mean continuous homogeneous isotropic Gaussian random field that is continuous with probability 1, such that

$$E\xi(x) = 0, \quad E\xi^2(x) = 1,$$