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Successive approximations to the optimal control of stochastic systems with after-effect. I

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Abstract—The method of successive approximations to the optimal control of stochastic differential equations is considered. Some examples are given.

In this review a method of successive approximations to the optimal control of stochastic differential equations with after-effect is described. This method is based on the following reasoning.

First, the class of functionals which integrally depends on the past history and is sufficiently smooth with respect to a current state of a system.

Second, every successive approximation to the optimal control is the optimal control for some auxiliary problem of control which is built in a special way.

The reader is referred to [1–4]. In [5] an application of modification of this method to the problems of optimal control in non-full data is shown.

1. A PROBLEM OF SYNTHESIS OF AN OPTIMAL CONTROL

In this section a formulation of the problem of synthesis of optimal control of stochastic systems with after-effect is given. The Bellman equation for the solution of this problem is deduced.

1.1. *Formulation of the problem*

Let $\{\Omega, \mathcal{F}, \mathbf{P}\}$ be a probability space with a flow of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$, $t \in [0,T]$, $\xi(t) \in \mathbb{R}^m$ be an $\mathcal{F}_t$-measurable Wiener process. Let H_0 be a set of piecewise-continuous functions $\varphi(s) \in \mathbb{R}^n$, $s \leqslant 0$, with the norm

$$\|\varphi\| = \sup_{s\leqslant 0} \left(\mathbf{E}|\varphi(s)|^2\right)^{1/2}.$$

The controllable stochastic system with after-effect is described by the Itô stochastic equation

$$\mathrm{d}x(t) = a(t, x_t, u)\,\mathrm{d}t + b(t, x_t, u)\,\mathrm{d}\xi(t) \tag{1.1}$$

with the initial condition $x_0 = \varphi_0 \in H_0$.

Here the n-dimensional vector functional $a(t,\varphi,u)$ and the matrix functional $b(t,\varphi,u)$ of dimension $(n \times m)$ are defined for $t \in [0,T]$, $y \in H_0$, and $u \in \mathbb{R}^l$. A control $u(t) = u(t, x_t)$ is a non-anticipating functional $u(t,\varphi)$ which is measurable with respect

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