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On stochastic equation describing the one-sided moving average process and minimax estimation problem

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Abstract—The problem of optimal linear estimation of the transformation

$$A\xi = \int_0^\infty \langle a(t), \xi(t)\rangle \, \mathrm{d}t$$

of a stationary stochastic process $\xi(t)$ with values in a Hilbert space from observations $\xi(t)$ as $t \leqslant 0$ is considered. The minimax spectral characteristics of the optimal estimate of the transformation $A\xi$ and the least favourable spectral densities for the various classes of densities are found.

Denote by X a separable Hilbert space with inner product $\langle x, y\rangle$ and an orthonormal basis $\{e_k\colon\ k = 1, 2, \ldots\}$. A stochastic process $\xi(t)$ with values in X is stationary, if its components $\xi_k = \langle \xi(t), e_k\rangle$ are square-mean continuous and satisfy the conditions from [1, 2]:

$$\mathbf{E}\xi_k(t) = 0, \qquad \mathbf{E}\|\xi(t)\|_X^2 = \sum_{k=1}^{\infty} \mathbf{E}|\xi_k(t)|^2 < \infty,$$
$$\mathbf{E}\xi_k(t)\bar{\xi}_j(s) = \langle B(t-s)e_k, e_j\rangle, \qquad k, j = 1, 2, \ldots .$$

The correlation function $B(t)$ of such a process is a weakly continuous operator-valued function on X. The correlation operator $B = B(0)$ is nuclear:

$$\sum_{k=1}^{\infty} \langle Be_k, e_j\rangle = \mathbf{E}\|\xi(t)\|_X^2 < \infty.$$

The stochastic process $\xi(t)$ has the spectral density $f(\lambda)$ if the correlation function $B(t)$ can be represented in the form

$$\langle B(t)e_k, e_j\rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{it\lambda} \langle f(\lambda)e_k, e_j\rangle \, \mathrm{d}\lambda, \qquad k, j = 1, 2, \ldots .$$

For amost all $\lambda \in \mathbb{R}^1$ the spectral density $f(\lambda)$ is a nuclear operator on X and its nuclear norm is integrable:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{k=1}^{\infty} \langle f(\lambda)e_k, e_k\rangle \, \mathrm{d}\lambda = \sum_{k=1}^{\infty} \langle B(0)e_k, e_k\rangle = \mathbf{E}\|\xi(t)\|_X^2 < \infty.$$

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