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Successive approximations to the optimal control of stochastic systems with after-effect. II

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Abstract—This paper is a continuation of [1]. The method of successive approximations to the optimal control of stochastic differential equations with after-effect is considered.

4. SYSTEMS WITH A NOISE UNDER CONTROL

In this section the problem of an approximate synthesis for the system with a small noise under control is considered. The Bellman equation and the optimal control which define an exact solution of the problem are given. By means of the algorithm developed in the previous sections the successive approximations to the optimal control are constructed.

4.1. *Formulation of the problem*

We consider the controllable system with random obstacles under control

$$\begin{aligned} dx_\varepsilon^u(t) &= \left(B(t) + \sqrt{\varepsilon}\sum_{i=1}^{N} \sigma_i\big(t, x_{\varepsilon t}^u\big)\dot{\xi}_i(t)\right) u\,dt + d\eta(t), \\ x_0 &= y_0 \in H_0. \end{aligned} \tag{4.1}$$

Here a process $\eta(t)$ is the same as in (3.1). Denote by $\xi_i(t)$, $i = 1, \dots, N$, independent scalar standard Wiener processes which do not depend on $\eta(t)$, the $(n \times l)$ matrices $B(t)$ and $\sigma_i(t, \varphi)$, $i = 1, \dots, N$, being uniformly bounded.

We show that the algorithm of construction of the approximate synthesis of the optimal control is valid for the problem (4.1), (3.3). This algorithm is analogous to that described above.

4.2. *The Bellman equation and the optimal control*

In this case the Bellman equation has the form

$$\begin{aligned} \mathcal{L}_0 V_\varphi(t,x) + x' N_2(t) x &= \frac{1}{4}\nabla V_\varphi'(t,x) B(t)(N_1(t) + \varepsilon R(V))^{-1} B'(t) \nabla V_\varphi(t,x), \\ V(T,\varphi) &= \varphi'(0) N_0 \varphi(0). \end{aligned} \tag{4.2}$$

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