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SECOND ORDER ASYMPTOTIC REPRESENTATION OF STOCHASTIC APPROXIMATION TYPE ESTIMATORS

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Abstract. For a smooth score function second order asymptotic representations of two different stochastic approximation type estimators are derived and the same is used to compare the recursive M-estimator related to the Robbins-Monro stochastic approximation procedure with the estimator which on the n-step is defined as the one step version M-estimator, where the estimator from the previous step is used as a preliminary one.

1 Introduction

Let $X_1, X_2, \ldots$ be independent identically distributed random variables (i. i. d. r. v.) with a distribution function (d. f.) $F(x, \theta_0)$, where $\theta_0 \in \Theta \subseteq R_1$ is an unknown parameter. Moreover, we assume that the parameter space Θ is an open interval.

Let $\psi: R_1 \times \Theta \to R_1$ be a function such that the function

$$\lambda(\theta) = \int \psi(x, \theta)\, dF(x, \theta_0) < +\infty \qquad \text{for all } \theta \in \Theta \tag{1.1}$$

and has a unique zero at $\theta = \theta_0$.

The following two types of estimators will be studied. *The recursive M-estimator* $\left\{\tilde{\theta}_k(\psi)\right\}_k$ *of θ_0 generated by the score function ψ* is defined as follows

$$\begin{aligned} \tilde{\theta}_{k+1}(\psi) &= \tilde{\theta}_k(\psi) - \frac{1}{(k+1)\tilde{\gamma}_k(\psi)}\, \psi\left(X_{k+1}, \tilde{\theta}_k(\psi)\right) \\ &\qquad\qquad \text{if the r.h.s. belongs to } \Theta \\ &= \tilde{\theta}_k(\psi) \qquad \text{otherwise,} \end{aligned} \tag{1.2}$$

$$\tilde{\gamma}_k(\psi) = \tilde{\gamma}_k^0(\psi) \qquad \text{if } (\log k)^{-1} \le |\tilde{\gamma}_k^0(\psi)| \le \log k \tag{1.3}$$

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