

Contents

Preface — vii

Notation — ix

0 Introduction — 1

1 Preliminaries — 13

- 1.1 Simple transformations and examples — 13
- 1.1.1 Dirac-type systems as a subclass of canonical systems — 13
- 1.1.2 Schrödinger systems as a subclass of canonical systems — 18
- 1.1.3 Gauge transformations of the Dirac systems — 19
- 1.2 S -nodes and Weyl functions — 22
- 1.2.1 Elementary properties of S -nodes — 22
- 1.2.2 Continual factorization — 24
- 1.2.3 Canonical systems and representation of the S -nodes — 27
- 1.2.4 Asymptotics of the Weyl functions, a special case — 30
- 1.2.5 Factorization of the operators S — 36
- 1.2.6 Weyl functions of Dirac and Schrödinger systems — 38

2 Self-adjoint Dirac system: rectangular matrix potentials — 44

- 2.1 Square matrix potentials: spectral and Weyl theories — 45
- 2.1.1 Spectral and Weyl functions: direct problem — 45
- 2.1.2 Spectral and Weyl functions: inverse problem — 48
- 2.2 Weyl theory for Dirac system with a rectangular matrix potential — 49
- 2.2.1 Direct problem — 49
- 2.2.2 Direct and inverse problems: explicit solutions — 56
- 2.3 Recovery of the Dirac system: general case — 61
- 2.3.1 Representation of the fundamental solution — 62
- 2.3.2 Weyl function: high energy asymptotics — 66
- 2.3.3 Inverse problem and Borg–Marchenko-type uniqueness theorem — 69
- 2.3.4 Weyl function and positivity of S — 73

3 Skew-self-adjoint Dirac system: rectangular matrix potentials — 79

- 3.1 Direct problem — 80
- 3.2 The inverse problem on a finite interval and semiaxis — 83
- 3.3 System with a locally bounded potential — 94

4 Linear system auxiliary to the nonlinear optics equation — 101

- 4.1 Direct and inverse problems — 102

4.1.1	Bounded potentials —	102
4.1.2	Locally bounded potentials —	106
4.1.3	Weyl functions —	115
4.1.4	Some generalizations —	117
4.2	Conditions on the potential and asymptotics of generalized Weyl (GW) functions —	118
4.2.1	Preliminaries. Beals–Coifman asymptotics —	118
4.2.2	Inverse problem and Borg–Marchenko-type result —	120
4.3	Direct and inverse problems: explicit solutions —	123
5	Discrete systems —	126
5.1	Discrete self-adjoint Dirac system —	126
5.1.1	Dirac system and Szegő recurrence —	127
5.1.2	Weyl theory: direct problems —	130
5.1.3	Weyl theory: inverse problems —	138
5.2	Discrete skew-self-adjoint Dirac system —	142
5.3	GBDT for the discrete skew-self-adjoint Dirac system —	156
5.3.1	Main results —	157
5.3.2	The fundamental solution —	160
5.3.3	Weyl functions: direct and inverse problems —	164
5.3.4	Isotropic Heisenberg magnet —	171
6	Integrable nonlinear equations —	177
6.1	Compatibility condition and factorization formula —	178
6.1.1	Main results —	178
6.1.2	Proof of Theorem 6.1 —	179
6.1.3	Application to the matrix “focusing” modified Korteweg-de Vries (mKdV) —	181
6.1.4	Second harmonic generation: Goursat problem —	185
6.2	Sine-Gordon theory in a semistrip —	188
6.2.1	Complex sine-Gordon equation: evolution of the Weyl function and uniqueness of the solution —	189
6.2.2	Sine-Gordon equation in a semistrip —	193
6.2.3	Unbounded solutions in the quarter-plane —	207
7	General GBDT theorems and explicit solutions of nonlinear equations —	210
7.1	Explicit solutions of the nonlinear optics equation —	210
7.2	GBDT for linear system depending rationally on z —	212
7.3	Explicit solutions of nonlinear equations —	221

8	Some further results on inverse problems and generalized Bäcklund-Darboux transformation (GBDT) — 230
8.1	Inverse problems and the evolution of the Weyl functions — 230
8.2	GBDT for one and several variables — 234
9	Sliding inverse problems for radial Dirac and Schrödinger equations — 242
9.1	Inverse and half-inverse sliding problems — 242
9.1.1	Main definitions and results — 242
9.1.2	Radial Schrödinger equation and quantum defect — 248
9.1.3	Dirac equation and quantum defect — 252
9.1.4	Proofs of Theorems 9.10 and 9.14 — 256
9.1.5	Dirac system on a finite interval — 257
9.2	Schrödinger and Dirac equations with Coulomb-type potentials — 259
9.2.1	Asymptotics of the solutions: Schrödinger equation — 260
9.2.2	Asymptotics of the solutions: Dirac system — 261
Appendices — 265	
A	General-type canonical system: pseudospectral and Weyl functions — 267
A.1	Spectral and pseudospectral functions — 268
A.1.1	Basic notions and results — 268
A.1.2	Description of the pseudospectral functions — 272
A.1.3	Potapov's inequalities and pseudospectral functions — 283
A.1.4	Description of the spectral functions — 290
A.2	Special cases — 297
A.2.1	Positivity-type condition — 297
A.2.2	Continuous analogs of orthogonal polynomials — 301
B	Mathematical system theory — 304
C	Krein's system — 306
D	Operator identities corresponding to inverse problems — 308
D.1	Operator identity: the case of self-adjoint Dirac system — 309
D.2	Operator identity for skew-self-adjoint Dirac system — 312
D.3	Families of positive operators — 313
D.4	Semiseparable operators S — 314
D.5	Operators with D -difference kernels — 317
E	Some basic theorems — 320
Bibliography — 323	
Index — 339	