

Contents

| | | |
|----------|-------------------------------------|----|
| 1 | Knots and isotopies | 1 |
| 1.A | Knots | 1 |
| 1.B | Equivalence of knots | 4 |
| 1.C | Knot projections | 9 |
| 1.D | Global geometric properties | 12 |
| 1.E | History and sources | 14 |
| 1.F | Exercises | 14 |
| 2 | Geometric concepts | 16 |
| 2.A | Geometric properties of projections | 16 |
| 2.B | Seifert surfaces and genus | 19 |
| 2.C | Companion knots and product knots | 21 |
| 2.D | Braids, bridges, plats | 23 |
| 2.E | Slice knots and algebraic knots | 26 |
| 2.F | History and sources | 28 |
| 2.G | Exercises | 28 |
| 3 | Knot groups | 30 |
| 3.A | Homology | 30 |
| 3.B | Wirtinger presentation | 32 |
| 3.C | Peripheral system | 40 |
| 3.D | Knots on handlebodies | 44 |
| 3.E | Torus knots | 48 |
| 3.F | Asphericity of the knot complement | 50 |
| 3.G | History and sources | 51 |
| 3.H | Exercises | 52 |

| | | |
|----------|---------------------------------------------------------|-----|
| 4 | Commutator subgroup of a knot group | 54 |
| 4.A | Construction of cyclic coverings | 54 |
| 4.B | Structure of the commutator subgroup | 57 |
| 4.C | A lemma of Brown and Crowell | 59 |
| 4.D | Examples and applications | 62 |
| 4.E | Commutator subgroups of satellites | 64 |
| 4.F | History and sources | 68 |
| 4.G | Exercises | 68 |
| 5 | Fibered knots | 71 |
| 5.A | Fibration theorem | 71 |
| 5.B | Fibered knots | 74 |
| 5.C | Applications and examples | 77 |
| 5.D | History and sources | 84 |
| 5.E | Exercises | 84 |
| 6 | A characterization of torus knots | 85 |
| 6.A | Results and sources | 85 |
| 6.B | An elementary proof of Theorem 6.1 | 88 |
| 6.C | Remarks on the proof | 93 |
| 6.D | History and sources | 96 |
| 6.E | Exercises | 96 |
| 7 | Factorization of knots | 97 |
| 7.A | Composition of knots | 97 |
| 7.B | Uniqueness of the decomposition into prime knots: proof | 102 |
| 7.C | Fibered knots and decompositions | 106 |
| 7.D | History and sources | 108 |
| 7.E | Exercises | 109 |
| 8 | Cyclic coverings and Alexander invariants | 111 |
| 8.A | Alexander module | 111 |
| 8.B | Infinite cyclic coverings and Alexander modules | 112 |
| 8.C | Homological properties of C_∞ | 119 |
| 8.D | Alexander polynomials | 122 |
| 8.E | Finite cyclic coverings | 132 |
| 8.F | History and sources | 137 |
| 8.G | Exercises | 137 |

| | | |
|-----------|------------------------------------------------------------|-----|
| 9 | Free differential calculus and Alexander matrices | 140 |
| 9.A | Regular coverings and homotopy chains | 140 |
| 9.B | Fox differential calculus | 142 |
| 9.C | Calculation of Alexander polynomials | 144 |
| 9.D | Alexander polynomials of links | 149 |
| 9.E | Alexander–Conway polynomial | 152 |
| 9.F | Finite cyclic coverings again | 155 |
| 9.G | History and sources | 158 |
| 9.H | Exercises | 158 |
| 10 | Braids | 161 |
| 10.A | The classification of braids | 161 |
| 10.B | Normal form and group structure | 169 |
| 10.C | Configuration spaces and braid groups | 174 |
| 10.D | Braids and links | 179 |
| 10.E | History and sources | 188 |
| 10.F | Exercises | 189 |
| 11 | Manifolds as branched coverings | 191 |
| 11.A | Alexander’s theorem | 191 |
| 11.B | Branched coverings and Heegaard diagrams | 197 |
| 11.C | History and sources | 206 |
| 11.D | Exercises | 207 |
| 12 | Montesinos links | 208 |
| 12.A | Schubert’s normal form of knots and links with two bridges | 208 |
| 12.B | 4-Plats (Viergeflechte) | 213 |
| 12.C | Alexander polynomial and genus of a knot with two bridges | 218 |
| 12.D | Classification of Montesinos links | 222 |
| 12.E | Symmetries of Montesinos links | 230 |
| 12.F | History and sources | 236 |
| 12.G | Exercises | 236 |
| 13 | Quadratic forms of a knot | 238 |
| 13.A | The quadratic form of a knot | 238 |
| 13.B | Computation of the quadratic form of a knot | 247 |
| 13.C | Alternating knots and links | 253 |
| 13.D | Comparison of different concepts and examples | 258 |

| | | |
|-----------|---------------------------------------------------------------------------------|------------|
| 13.E | History and sources | 264 |
| 13.F | Exercises | 264 |
| 14 | Representations of knot groups | 266 |
| 14.A | Metabelian representations | 266 |
| 14.B | Homomorphisms of \mathcal{G} into the group of motions of the Euclidean plane | 272 |
| 14.C | Linkage in coverings | 279 |
| 14.D | Periodic knots | 285 |
| 14.E | History and sources | 297 |
| 14.F | Exercises | 297 |
| 15 | Knots, knot manifolds, and knot groups | 301 |
| 15.A | Examples | 301 |
| 15.B | Property P for special knots | 304 |
| 15.C | Prime knots and their manifolds and groups | 315 |
| 15.D | Groups of product knots | 329 |
| 15.E | History and sources | 332 |
| 15.F | Exercises | 333 |
| 16 | Bridge number and companionship | 334 |
| 16.A | Seifert surfaces for satellites | 334 |
| 16.B | Companions of order one. | 337 |
| 16.C | Bridge number and height functions | 339 |
| 16.D | History and sources | 352 |
| 16.E | Exercises | 352 |
| 17 | The 2-variable skein polynomial | 353 |
| 17.A | Construction of a trace function on a Hecke algebra | 353 |
| 17.B | The HOMFLY-PT polynomial | 359 |
| 17.C | History and sources | 364 |
| 17.D | Exercises | 364 |
| A | Algebraic theorems | 365 |
| B | Theorems of 3-dimensional topology | 371 |
| C | Table | 375 |
| C.1 | Table | 376 |
| D | Knot projections 0_1-9_{49} | 384 |

| | |
|---------------------|-----|
| References | 387 |
| Author index | 407 |
| Glossary of Symbols | 411 |
| Index | 413 |