

NEWS

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Approximations and Endomorphism Algebras of Modules

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Open problem progress

Page	Problem no.	Comment
130	1	A positive answer for the cotorsion pairs (A, B) such that B is closed under direct limits (and hence for all tilting cotorsion pairs) is given in [A5].
251	2	A positive solution is given in [A5]: the class of all flat Mittag-Leffler modules is precovering, iff R is a right perfect ring. [A5] contains further results of this type, notably on locally Baer modules over hereditary artin algebras.
381	2	By [A4], for each right R -module M , we have $\text{Prod}(M^*) = \text{Prod}(\mathcal{N})$ where \mathcal{N} is a set of indecomposable direct summands in M^* . It follows that the answer to Problem 2 is positive for all cotilting modules of cofinite type (and hence for all cotilting modules over commutative noetherian rings, and all 1-cotilting modules over left noetherian rings).
398	1	The first part of the problem is solved in [A9]: for each characteristic sequence \bar{X} , there is a unique minimal cotilting module $C_{\bar{X}}$ inducing the cotilting class $\mathcal{C}_{\bar{X}}$. This module is constructed explicitly in [A9] by an iteration of injective covers. (Here, minimality refers to the fact that $C_{\bar{X}}$ is a direct summand in each cotilting module equivalent to it.) However, the construction of the tilting module $T_{\bar{X}}$ remains open in general.
716	1	This problem has been answered as part of [382], see Remark 24.56. The suggested new approach avoids the triple modules from Sections 24.4 and 24.5 and separates the construction into a combinatorial component and an algebraic component. The advantage of this approach is that the construction becomes easily adaptable to other situations by adjusting the algebraic component while keeping the entire combinatorial machinery unchanged. This method has been successfully applied to the construction of \mathfrak{S}_n -free $E(R)$ -algebras in [382] and to the construction of \mathfrak{S}_n -free separable groups with prescribed endomorphism rings in [301].

716	2	This problem has been answered as part of [382]. Furthermore, it was suggested to consider the Easy Black Box (Proposition 19.2) as the combinatorial backbone of any future Black Box construction. The construction of \mathbb{S}_n -free structures reduces to the combination of an algebraic version of this Easy Black Box (not unlike the General Black Box 19.23) with a particularly strong Step Lemma.
831	3	This problem has been answered as part of [382].

Corrections and updates

Page	Line	Comment
123	1	<i>Supplementary information:</i> Cotorsion pairs also play an important role in introducing model category structures, and hence computing cohomology of complexes of modules and quasi-coherent sheaves. For these applications of cotorsion pairs, we refer to [232], [392], and [A7].
202	1	Replace the reference [232, §4] with [A6, Theorem 1.4 and Corollary 1.5].
221	-10	Replace [349, 6.20 and Ex. 7G] with [358, 6.20 and Ex. 7G].
221	-7	Replace [349, 7.11 and 7.13] with [358, 7.11 and 7.13].
237	13	Replace \mathcal{C} with \mathcal{H} in the last subscript.
236 – 240		<i>Supplementary information:</i> This part of Section 10.2 can substantially be generalized using tilting theory. The basic idea is to replace the 0-tilting module R (which is \sum -pure-split, if and only if R is right perfect) with any non- \sum -pure-split n -tilting module T . The class of all flat Mittag-Leffler modules is thus generalized to the class of all locally T -free modules. Its non-deconstructibility, and non-precovering, can be proved for arbitrary (not necessarily countable) rings. For more details and applications, we refer to [A3], [A5], and [A6].
241	6	Replace \leq -increasing with \prec -increasing.
244	-8	Replace M with A in the pullback diagram.
297	-8	<i>Supplementary information:</i> For further aspects of tilting theory, we refer to [18] and [A1].
350	-13	Replace [349, Ex.4B] with [358, Ex.4B].
336	20	<i>Supplementary information:</i> Large tilting and cotilting modules over tame algebras of tubular type have recently been classified in [A2].
367	-1	<i>Supplementary information:</i> Cotilting modules can also be characterized as images of injective cogenerators of Grothendieck categories under certain tilting derived equivalences. This approach is developed in [A8].
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378	4 and 6	Replace the localization $C_{(m)}$ with the colocalization $C^{(m)} = \text{Hom}_R(R_{(m)}, C)$. The result holds also in the commutative noetherian setting. For more details, see [A10].
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751	11	Replace Srettawisarn with Petapirak.

Update of the bibliography

Reference	Update
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| [26] | Has appeared in J. reine angew. Math. 682 (2013), 1-48. |
| [79] | Has appeared in Proc. Amer. Math. Soc. 141 (2013), 1177-1183. |
| [144] | Add: see also reference [296]. |
| [227] | Has appeared in Bull. London Math. Soc. 44 (2012), 439-442. |
| [233] | To appear in Proc. Amer. Math. Soc. |
| [301] | Replace in preparation with submitted. |
| [304] | To appear in J. European Math. Soc. 17 (2014). |
| [382] | Update the reference as follows: D.Herden: <i>Constructing \aleph_k-free structures</i> , Habilitationsschrift, Univ. Duisburg-Essen 2013. |
| [525] | Has appeared in Forum Math. 24 (2012), 1091-1109. |
| [562] | Replace M.Srettawisarn with M.Petapirak. |

Additional bibliography

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|------|---|
| [A1] | L.Angeleri Hugel: <i>Infinite dimensional tilting theory, to appear in Advances in Representation theory of Algebras</i> , EMS Series of Congress Reports (eds. D. J. Benson, H. Krause, A. Skowronski). |
| [A2] | L.Angeleri Hugel, D.Kussin: <i>Large tilting modules and sheaves for canonical algebras</i> , preprint. |
| [A3] | D.Herbera: <i>Definable classes and Mittag-Leffler conditions</i> , to appear in Proc. 31st Ohio State - Denison Math. Conference. |
| [A4] | A.R.Mehdi, M.Prest: <i>Almost dual pairs and definable classes of modules</i> , preprint, available at arXiv:1304.4481v1 (http://arxiv.org/abs/1304.4481). |
| [A5] | J.Šaroch, L.Angeleri Hugel, J.Trlifaj: <i>Tilting approximations and Mittag-Leffler conditions</i> , preprint. |
| [A6] | A.Slavik, J.Trlifaj: <i>Approximations and locally free modules</i> , published ahead of print in Bull. London Math. Soc. (2013), DOI: 10.1112/blms/bdt069. |

- [A7] J. Šťovíček: *Exact model categories, approximation theory, and cohomology of quasi-coherent sheaves*, to appear in *Advances in Representation Theory of Algebras*, EMS Series of Congress Reports (eds. D. J. Benson, H. Krause, A. Skowronski).
- [A8] J. Šťovíček: *Derived equivalences induced by big cotilting modules*, preprint, available at arXiv:1308.1804v1 (<http://arxiv.org/abs/1308.1804>).
- [A9] J. Šťovíček, J. Trlifaj, D. Herbera: *Cotilting modules over commutative noetherian rings*, preprint, available at arXiv:1306.6788v1 (<http://arxiv.org/abs/1306.6788>).
- [A10] J. Trlifaj, S. Sahinkaya: *Colocalization and cotilting for commutative noetherian rings*, preprint, available at arXiv:1306.6234v1 (<http://arxiv.org/abs/1306.6234>).
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We apologize for the misprints listed on page 2 above. If you notice any other ones, please let us know at info@degruyter.com. Thank you in advance!

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