Long wavelength enhanced Kerr nonlinearity via Fano-type interference in semiconductor quantum wells

Abstract
The linear and nonlinear behaviors of susceptibility via tunable Fano-type interference, based on intersubband transitions in asymmetric double quantum wells (QWs) driven coherently by a probe laser field, are analyzed. It is shown that Kerr nonlinearity can be controlled competently by tuning the energy splitting of the two excited states (the coupling strength of the tunneling), i.e. Fano-type interference. This outcome may be used for optimizing and controlling long wavelength optical switching processes in QW solid-state systems, which is much more useful than that currently used in atomic systems because of its flexible design and the controllable interference strength.

Keywords
Fano interference • Kerr nonlinearity

PACS:
© Versita sp. z o.o.

1. Introduction

As is known, nowadays the long wavelength window is the standard optical communication band, in particular for long distance transmission systems. A long wavelength infrared (IR) photo detector based on electromagnetically induced transparency (EIT) that is suitable for operation around room temperature and in the THz range has been proposed by Zyaei et al. [1] It is known that strong light–matter interactions in nonlinear optical materials have a crucial role in all-optical switching and optical communication. Thus, it is very important that one can enhance the Kerr nonlinearity at long wavelengths. Kerr nonlinearity has been extensively studied both experimentally [2] and theoretically [3–6] in multi-level atomic media due to its wide range of applications, such as in quantum bit regeneration [7], and all-optical switching [8]. Kerr nonlinearity corresponds to the refractive part of the third-order susceptibility in optical media. The large enhancement of nonlinear susceptibilities with decreasing absorption has attracted tremendous interest in nonlinear optics at low light levels [9]. This work is based on quantum coherence and interference effects, which can produce some interesting phenomena such as electromagnetically induced transparency (EIT) [10], lasing without population inversion [11], phase control of group velocity [12] and controlling of optical bistability [13, 14], and so on. It is known that under the conditions of electromagnetically induced transparency it is possible to control the optical response and related absorption of weak laser light. This effect has been deeply studied in atomic physics [15–20] starting from its observation in sodium vapors [21] and has resulted in new possibilities for nonlinear optics and quantum information processing. As we know, it is easy to realize EIT in optically dense atomic media in the gas phase, but more difficult to observe EIT in solid-state media because of the short coherence times in solid-state systems [10]. However, some investigations have demonstrated EIT effects and ultraslow optical pulse propagation in semiconductor quantum well (SQW) structures [22–28]. The reason for these investigations is essentially that the phenomena in SQWs have many potentially significant applications in optoelectronics and solid-state quantum information science. Otherwise, devices based on intersubband transitions in SQWs have many natural advantages over atomic systems, such as large electric dipole moments due to the small effective electron mass, high nonlinear optical coeffi-
scent, great flexibility in device design through the choice of materials and structural dimensions, and the transition energies and dipoles, as well as the symmetries, can also be engineered as desired.

It is worth pointing out that Joshi and Xiao recently studied bistable behavior in a semiconductor quantum well that interacts with two electromagnetic fields, a strong field and a weak field, and demonstrated that the threshold for switching to the upper branch of the bistable curve can be reduced through the presence of quantum interference [29]. The bistable behaviors via tunable Fano-type interference in asymmetric semiconductor quantum wells with three-subband V configurations have also also studied [30, 31].

In our recent work, we analyzed the effect of quantum coherence and interference on linear and nonlinear susceptibility of an asymmetric coupled quantum well [32]. It was shown that giant Kerr nonlinearity can be obtained by a control field and electron tunneling. In another work, we showed that the Kerr nonlinearity can be enhanced at long wavelengths in quantum dot nanostructures [33]. We showed that electron tunneling in quantum dots has a crucial role in the enhancement of Kerr nonlinearity.

In this paper, for the first time in the absence of a controlling coupling field, long wavelength giant Kerr nonlinearity via tuning the coupling strength of the tunneling is demonstrated; and it is shown that Fano-type interference in asymmetric double quantum well structures using intersubband transitions can be obtained.

2. Model and equations

The double quantum well scheme under discussion here consists of two quantum wells that are separated by a narrow barrier, as shown in Fig. 1. The first subband (|a⟩) of the shallow well is resonant with the second subband (|b⟩) of the deep well (see Fig. 1a), and the strong coherent coupling via the thin barrier causes the levels to split into a doublet, levels |2⟩ and |3⟩ (see Fig. 1b). These levels arise from the mixing of the states |a⟩ and |b⟩, under the exactly resonant conditions. The splitting ωe on resonance is given by the coupling strength and can be controlled by adjusting the height and width of the tunneling barrier with an applied bias voltage [34]. A weak probe field with frequency ωp interacts on both the transitions |1⟩ ↔ |2⟩ and |1⟩ ↔ |3⟩ simultaneously with respective Rabi frequencies \( \Omega_{p1} = \mu_{12} E_p / 2 \hbar \) and \( \Omega_{p2} = \mu_{13} E_p / 2 \hbar \). \( E_p \) is the amplitude of the weak probe field and \( \mu_{ij} = \mu_{ji} \cdot \hat{e}_z \) is the dipole moment for the transition between levels \(|i⟩\) and \(|j⟩\) with \( \hat{e}_z \) being the unit polarization vector of the corresponding laser field. The dynamic response of the three-level QW system is described using the density matrix formalism. This method has been used in several experimental and theoretical papers [35–38]. With the usual phenomenological addition of relaxation processes, the elements of density matrix equations can be calculated. Thus, in the rotating wave approximation, the density matrix equations of motion for this system can be obtained as

\[
\rho_{12} = -\frac{\gamma_1}{2} \rho_{12} + i (\Delta_p - \frac{\omega_p}{2}) \rho_{12} + i \Omega_{p1} (\rho_{21} - \rho_{12}) + \frac{k}{2} \rho_{13} \\
\rho_{13} = -\frac{\gamma_3}{2} \rho_{13} + i (\Delta_p + \frac{\omega_p}{2}) \rho_{13} + i \Omega_{p3} (\rho_{31} - \rho_{13}) + \frac{k}{2} \rho_{23} \\
\rho_{23} = -\frac{\gamma_2}{2} \rho_{23} + i \omega_p \rho_{23} + i \Omega_{p2} (\rho_{32} - i \Omega_{p2} \rho_{13} - \rho_{23}) - \frac{k}{2} (\rho_{22} + \rho_{33}) \\
\rho_{22} = -\rho_{22} + i \Omega_{p2} (\rho_{12} - \rho_{21}) - \frac{k}{2} (\rho_{23} + \rho_{32}) \\
\rho_{33} = -\rho_{33} + i \Omega_{p3} (\rho_{13} - \rho_{31}) - \frac{k}{2} (\rho_{23} + \rho_{32})
\]

(1)

The following conditions are used for the above analysis: (a) the electron sheet density of the quantum well structure is such that electron–electron effects have only a very small influence on our results. Thus, the effects of electron–electron interactions are not included in our results; (b) it is assumed that all subbands have the same effective mass. Here, \( \omega_i = E_3 - E_2 \) is the energy splitting between the upper levels, given by the coherent coupling strength of the tunneling. \( \Delta_p = \omega_0 - \omega_p \) is the detuning between the frequency of the probe laser and the average transition frequency \( \omega_0 = (\omega_2 + \omega_1) / 2 \). The total decay rates are given by \( \gamma_{21} = \gamma_2 + \gamma_{dp2} \), \( \gamma_{31} = \gamma_3 + \gamma_{dp3} \), and \( \gamma_{32} = \gamma_3 + \gamma_{dp3} \). Here, \( \gamma_i \) are the spontaneous decay rates, and \( \gamma_{ij}^{dp} \) represents the coherence relaxation, determined by electron–electron interactions, interface roughness, and phonon scattering processes, and corresponding to the dephasing decay rate of the quantum coherence of \(|i⟩ ↔ |j⟩\). \( k = \sqrt{\gamma_2 \gamma_3} \) represents the mutual coupling of states |2⟩ and |3⟩ via the LO phonon decay; it describes the process in which a phonon is emitted by subband |2⟩ and is recaptured by subband |3⟩. If tunneling is present, these mutual coupling terms can be obtained, e.g., through an additional barrier next to the deeper well [34]. In this case, levels |2⟩ and |3⟩ are both the superpositions of the resonance states |a⟩ and |b⟩.

Since the latter (subband |b⟩) is strongly coupled to a continuum via a thin barrier, the decay from state |b⟩ to the continuum inevitably results in these two dependent decay pathways: from the excited doublet to the common continuum. That is to say, the two decay pathways are...
related: the decay from one of the excited doublets can strongly affect the neighboring transition, resulting in the interference characterized by those mutual coupling terms. Such interference is similar to the “decay-induced” coherence in atomic systems with two closely lying energy states. In the following numerical calculations the choices of the parameters are based on experimental results from reference [34]. The intensity of the Fano interference [34], defined by \( P = k \sqrt{y_{21} y_{31}} = \sqrt{y_{21} y_{31}} \), and the values \( P = 0 \) and \( P = 1 \) corresponds to no interference (there is no negligible coupling between \( y_{21} \), \( y_{31} \)). This means that Fano interference does not occur and perfect interference (no dephasing decay rates \( y_{i} = 0 \), respectively. It is worth noting that the above described parameter \( \rho \) is mainly controlled via the population decay rates \( y_{i} \) and dephasing decay rates \( y_{i} = y_{i}^{ph} \) present \( k \neq 0 \) but disappears when the cross coupling term is absent \( k = 0 \).

As is known, the response of the atomic medium to the weak probe field is governed by its polarization \( P = \epsilon_0(E_{F} X + E_{p} \chi)/2 \) with \( \chi \) being the susceptibility of the atomic medium. By performing a quantum average of the dipole moments over an ensemble of \( N \) atoms, it is found that \( P = N \rho_{31}\rho_{12} + \rho_{31}\rho_{13} + \rho_{32}\rho_{12} + \rho_{21}\rho_{12} \). In order to derive the linear and nonlinear susceptibility, the steady state solution of density matrix equations is needed. The density matrix elements can be expanded as \( \rho_{i} = \rho_{i}^{(0)} + \rho_{i}^{(1)} + \rho_{i}^{(2)} + \ldots \). The zeroth order solution of \( \rho_{12} \) will be identical, i.e., \( \rho_{12}^{(0)} = 1 \), and other elements are set to zero. The first and third-order susceptibilities \( \chi_{i}^{(1)} \) and \( \chi_{i}^{(3)} \) of the medium can be determined by coherence terms \( \rho_{31}^{(1)} \) and \( \rho_{31}^{(3)} \) in Eq. 2a and \( \rho_{31}^{(1)} \), \( \rho_{31}^{(3)} \) in Eq. 2b, respectively.

\[
\chi_{i}^{(1)} = \frac{2N}{\epsilon_0 \hbar \Omega_p} (\rho_{12}^{(1)} + \rho_{13}^{(1)} + \rho_{32}^{(1)}) \tag{2a}
\]

\[
\chi_{i}^{(3)} = \frac{2N}{\epsilon_0 \hbar \Omega_p} (\rho_{12}^{(3)} + \rho_{13}^{(3)} + \rho_{32}^{(3)}) \tag{2b}
\]

Substituting coherence terms \( \rho_{31} \) and \( \rho_{21} \) into Eq. 3 and assuming \( \rho_{31} = \rho_{21} = \rho \), the linear and nonlinear susceptibility are obtained as:

\[
\chi_{i}^{(1)} = \frac{2N}{\epsilon_0 \hbar \Omega} (\rho_{i}^{(1)} - 2\Delta_{p} + i(k - \frac{1}{2}(y_{31} + y_{21}))) \tag{3a}
\]

\[
\chi_{i}^{(3)} = \frac{2N}{\epsilon_0 \hbar Z} \left[ \frac{i\gamma_{31}}{2} + \frac{\omega_{l}}{2} \right] \left[ (\rho_{22}^{(1)} - \rho_{12}^{(1)}) + (\rho_{23}^{(1)} - \rho_{13}^{(1)}) \right] - \frac{i}{k} \left[ \frac{\gamma_{21}}{2} - \frac{\omega_{l}}{2} \right] \left[ (\rho_{33}^{(1)} - \rho_{11}^{(1)}) + (\rho_{32}^{(1)} - \rho_{12}^{(1)}) \right] \tag{3b}
\]

where

\[
Z = \frac{1}{2} (\Delta_{p} - \frac{\omega_{l}}{2}) \Delta_{p} + \frac{i}{2} (y_{21} + y_{31}) \Delta_{p} + \frac{i}{4} (y_{31} - y_{21})
\]

\[
+ \frac{y_{21} y_{31}}{4} - \frac{k^{2}}{4} \tag{4}
\]

In these expressions we set \( \Omega_{p1} = \Omega_{p2} = \Omega_{p} \), where \( \Omega_{p} \) are to be real. Here, \( N \) is the atomic number density matrix in the medium. The linear dispersion and absorption are proportional to the real and imaginary parts of \( \chi_{i}^{(1)} \), while, the real and imaginary parts of \( \chi_{i}^{(3)} \) determine the nonlinear dispersion i.e. Kerr nonlinearity and the nonlinear absorption.

3. Results and discussion

Now we give some numerical studies under the steady-state conditions, as shown in Fig. 2, 3, 4, 5, 6. In the following numerical calculations, we assume that all sub-bands have the same effective mass and the electron-electron effects have only a very small influence on our results. We can know the effect of Fano interference and energy splitting on the linear and nonlinear optical properties of this structure. The following is focused on enhancing Kerr nonlinearity with nonabsorption (including the linear and nonlinear absorptions). First of all, the linear and nonlinear susceptibility is analyzed for \( \omega_{l} = 17.6 \) meV, \( y_{2} = 5.6 \) meV, \( y_{1} = 7.0 \) meV, \( y_{21} = 1.5 \) meV, \( y_{31} = 2.3 \) meV, and \( y_{32} = 1.9 \) meV. The linear and nonlinear susceptibility for the above parameters is shown in Fig. 2(a) and (b). It is found that Kerr nonlinearity is seen (dashed line in Fig. 2(b)), accompanied by strong linear and nonlinear absorption (solid lines of Fig. 2(a) and (b)). In this case, the results are not suitable for applications in nonlinear optical phenomenon, such as all-optical switching. The effect of energy splitting on linear and nonlinear susceptibility is shown in Fig. 3(a) and (b). It is found that by increasing the value of energy splitting to 21.3 meV, the refractive part of third-order susceptibility is enhanced (dashed line in Fig. 3(b)) and also the linear and nonlinear absorption (solid lines of Fig. 2(a) and (a)) are reduced. It can be clearly seen that the maximal Kerr nonlinearity at \( \omega_{l} = 21.3 \) meV is three times that at \( \omega_{l} = 17.6 \) meV. In addition the
Fig 1. (a) Energy level diagram of a double quantum well structure. It consists of two quantum wells and a collector region separated by thin tunneling barriers. (b) Due to the strong coherent coupling via the thin barrier, the levels split into a doublet \( |2\rangle \) and \( |3\rangle \) that are coupled to a continuum by a thin tunneling barrier adjacent to the deep well. Splitting occurs between the two upper levels \( \omega_s \) (given by the coherent coupling strength) and the weak probe laser \( \omega_p \).

maximal Kerr nonlinearity enters the reduced linear absorption window and therefore the corresponding linear absorption becomes negligible. The corresponding transmission coefficient is shown in Fig. 4. It is obtained that by enhancing the energy splitting the transmission coefficient is also enhanced and reaches approximately 1 for \( \omega_s = 21.3 \text{ meV} \) (solid line). Now we provide a qualitative explanation for the above numerical results. It can be seen from Eq. 3b that the density matrix element \( \rho_{23}^{(2)} \) is now an extra term introduced by energy splitting \( \omega_s \) and mutual coupling of states \( |2\rangle \) and \( |3\rangle \) (k). These terms accordingly make the third-order susceptibility acquire additional terms associated with \( \omega_s \) and \( k \). As mentioned above, the splitting on resonance is given by the coupling strength and can be controlled by adjusting the height and width of the tunneling barrier. Therefore, the behavior of linear and nonlinear susceptibility can be tuned by appropriately adjusting the tunneling barrier. The effects of the strength or values of the interference on linear absorption can be clearly seen from Fig. 5(a). It can be seen that the linear absorption is approximately vanishes as we go from \( P = 0.46 \) to \( P = 0.77 \). The effects of the strength or the value of the interference on the Kerr nonlinearity is shown in Fig. 5(b). It can be seen that the Kerr nonlinearity is enhanced about six times as we go from \( P = 0.46 \) to \( P = 0.77 \). A more interesting result is obtained for the nonlinear absorption spectrum. It is found that by increasing the value of Fano interference, the nonlinear absorption vanishes and is converted to the nonlinear gain. On the hand, it is found that in the case of \( P = 0.77 \), the enhanced Kerr nonlinearity is located at \( \lambda = 1.55 \mu m \) in the nonlinear gain and zero linear absorption region. So in this case, we achieve giant Kerr nonlinearity accompanied by zero linear absorption and negative nonlinear absorption. Finally, the transmission coefficient for the above parameters is shown in Fig 6. It is found that for \( P = 0.77 \) the transmission coefficient (solid line) approximately reaches 1. This result has a
Fig 3. Linear and nonlinear susceptibility versus probe wavelength. (a) Linear absorption (solid line) and linear dispersion (dashed line). (b) Nonlinear absorption (solid line) and Kerr nonlinearity (dashed line). $\omega_1 = 21.3 \text{ meV}$ and other parameters are the same as in Fig. 2.

Fig 4. The transmission coefficient versus probe wavelength. The dashed line corresponds to $\omega_1 = 17.6 \text{ meV}$, the solid line corresponds to $\omega_1 = 21.3 \text{ meV}$. Other parameters are the same as in Fig. 2.

Fig 5. (a) Linear absorption, (b) Kerr nonlinearity and (c) nonlinear absorption versus probe wavelength. The solid lines correspond to $P = 0.77$ and dashed lines correspond to $P = 0.46$. $\omega_1 = 18.6 \text{ meV}$ and the other parameters are the same as in Fig. 2.

Fig 6. The transmission coefficient versus probe wavelength. The dashed line corresponds to $P = 0.46$; the solid line corresponds to $P = 0.77$. Other parameters are the same as in Fig. 5.
good agreement with previous results that were obtained. Finally we emphasize this note that enhancing the Kerr nonlinearity with reduced linear absorption is very important for the all-optical switching, frequency modulation and relevant fields.

4. Conclusion

In conclusion, we have illustrated the linear and nonlinear behaviors of susceptibility in a three-subband QW system driven by a coherent probe field. It is found that through the energy splitting of two excited states (the coupling strength of the tunneling), Fano-type interference can enhance the Kerr nonlinearity and reduce linear and nonlinear absorption. Our calculations also provide a guideline for the optimal design of QW systems to achieve very fast and low-threshold all-optical switches [39–43] in such semiconductor systems, which is much more practical than that developed in atomic systems because of its flexible design and the controllable interference strength.

References