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Heisenberg-type ferromagnetic films with a sandwich structure

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Abstract: Green's function theory is applied to an alternating two-element multilayer ferromagnetic film. The Hamiltonian is based on the Heisenberg model with a second-order uniaxial single-ion anisotropy and with the interaction between the layer in the interfaces. Magnetization and the phase transition in such system are derived for several sets of material parameters.
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1 Introduction

The magnetic properties of thin films are currently the object of intense research efforts. The motivation for this interest is the possibility to produce films and multilayers with perpendicular magnetic anisotropy. The topic of perpendicular magnetic anisotropy is a very challenging one, since the magnetostatic dipolar interactions always prefer in-plane orientation of the easy axis of magnetization. Since the pioneering experimental work of Gradmann [1] and the theoretical predictions of Gay and Richter [2], much effort has been spent investigating Fe, Co, and Ni films grown on noble-metal substrates. In

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these systems a reorientation transition between perpendicular and in-plane direction was observed at the transition temperatureT_R below the Curie temperature [3]. The existence of perpendicular magnetization is determined by the combined effects of all the magnetic anisotropies present. These anisotropies include shape, surface, interface, and crystalline anisotropies, strain-induced magnetoelastic anisotropy, and anisotropies due to roughness and atomic mixing at the interface [4]. An important goal in the research on thin films and multilayers is to understand which anisotropies are important in determining the easy axes of magnetism.

Sato *et al.* [5] prepared a class of Tb-Fe films by a deposition of ultra-thin alternate layers of Tb and Fe and reported that Tb-Fe thin films exhibit superior magnetic characteristics such as large uniaxial anisotropy. The most important origin of the large magnetic uniaxial anisotropy can be attributed to the Tb-Fe pairs aligned perpendicular to the films. The effect of the interaction of Tb-Fe spin pairs at the interface on the magnetic properties of the Tb-Fe multilayer structure has been studied within the Green's function method [6] with conclusion that the order parameter changed with the position of the atomic planes in the multilayer films but the critical temperature of atomic planes is all the same. This numerical calculation agrees with the results of magnetic measurements.

The motivation for this work has been to study the effect of the uniaxial single-ion anisotropy and exchange anisotropy on the magnetic properties of the artificially layered structure based on the model crystal that consists of alternating layers of two different ferromagnets F and G.

The paper is organized as follows. In section 2 the Green's function formalism is outlined. The eigenvector method is then presented for the considered sandwich structure. Section 3 deals with the results. First we investigate the effect of the anisotropy of the exchange interaction (parameter D) at the interface for different values of thickness of one ferromagnet when the parameters of the single-ion anisotropy (parameters K_F, K_G) in both materials are fixed. Secondly, we discuss the critical parameter D_C and the critical ratio $\eta_C = (J_F/J_G)_C$ of the exchange parameters J_F, J_G in both materials as a function of the single-ion anisotropy parameters K_F, K_G.

2 Theory

The Hamiltonian we use in this investigation consists of a ferromagnetic isotropic Heisenberg exchange interaction with strength $J_{ij} > 0$, an uniaxial exchange anisotropy with strength D_{ij} due to the exchange interaction of the spin pairs at the interface (for simplicity, only the nearest neighbor coupling is taken into account) and a second order single-ion anisotropy with strength $K_i > 0$, which favors an orientation of the spin in z direction:

$$H = -\frac{1}{2}\sum_{<ij>} J_{ij}(S_i^- S_j^+ + S_i^z S_j^z) - \frac{1}{2}\sum_{<ij>} D_{ij} S_i^z S_j^z - \sum_i K_i (S_i^z)^2, \tag{1}$$

where J_{ij} are J_F or J_G if i and j are pairs of nearest neighbours within the ferromagnetic film F or the ferromagnetic film G, respectively, D_{ij} is D if i and j are spin pairs of nearest neighbours at the interface and K_i is K_F and K_G in the layers of the film F and G, respectively. Here the notation $S_j^{\pm} = S_j^x \pm i S_j^y$ is introduced. We assume that the film F has a simple cubic structure and the film G is monolayer with a square structure. Both lattices have the same lattice constant.

To theoretically investigate the influence of the single-ion anisotropy and the exchange anisotropy at the interface on the phase transition in the above mentioned system, we use the eigenvector method (EVM) [7] for the calculation of expectation values in terms of the eigenvalues and eigenvectors of the matrix of equations of motion for the set of Green's functions.

In order to treat the problem for general spin S, we need the following Green's function

$$G_{ij}^m(\omega) = \left\langle\left\langle S_i^+; (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega , \tag{2}$$

where m is zero or positive integer, necessary for dealing with higher spin values S: $m \leq 2S-1$. The exact equations of motion are

$$\omega G_{ij}^{(m)}(\omega) = A_{ij}^{(m)}\delta_{ij} + \left\langle\left\langle [S_i^+; H]; (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega \tag{3}$$

with the inhomogeneities

$$A_{ij}^{(m)} = \left\langle [S_i^+, (S_j^z)^m S_j^-] \right\rangle, \tag{4}$$

where $\langle \cdots \rangle = Tr(\cdots e^{-\beta H})/Tre^{-\beta H}$ denotes the thermodynamic expectation value, the brackets $[\cdots]$ denote the commutator and δ_{ij} is the Kronecker's symbol. After calculation of the commutator $[S_i^+; H]$, we obtain the following equations of motion

$$\begin{aligned}\omega G_{ij}^{(m)}(\omega) = A_{ij}^{(m)}\delta_{ij} + \sum_{k\neq i} J_{ik}\left(\left\langle\left\langle S_k^z S_i^+; (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega - \left\langle\left\langle S_i^z S_k^+; (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega\right) + \\ + K_i \left\langle\left\langle (S_i^+ S_i^z + S_i^z S_i^+); (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega + D\sum_{k\neq i}\left\langle\left\langle S_k^z S_i^+; (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega .\end{aligned} \tag{5}$$

After solving these equations the magnetization can be determined from the Green's functions *via* the spectral theorem. A solution is possible by establishing a closed system of equations by decoupling the higher-order Green's functions on the right hand side. We stay here at the level of first-order Green's functions. For the exchange-interaction term, we apply a Tyablikov- (or Random Phase Approximation (RPA)) decoupling [8, 9]:

$$\left\langle\left\langle (S_k^z S_i^+ - S_i^z S_k^+); (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega \cong \langle S_k^z \rangle\, G_{ij}^{(m)}(\omega) - \langle S_i^z \rangle\, G_{kj}^{(m)}(\omega). \tag{6}$$

The terms from the single-ion anisotropy have to be decoupled differently, because RPA leads to unphysical results; *e.g.* for spin $S = 1/2$ the terms due to the single-ion anisotropy do not vanish in RPA, as they should do. We choose the Anderson-Callen decoupling procedure [10] which gives good results [11, 12] for the magnetization if the anisotropy parameters K_F, K_G are much smaller than the exchange coupling.

The Anderson-Callen decoupling yields

$$\left\langle\left\langle (S_i^+ S_i^z + S_i^z S_i^+); (S_j^z)^m S_j^- \right\rangle\right\rangle_\omega \cong 2\langle S_i^z \rangle \left(1 - \frac{1}{2S^2}[S(S+1) - \left\langle (S_i^z)^2 \right\rangle]\right) G_{ij}^{(m)}(\omega). \tag{7}$$

Due to translation symmetry, $G_{ij}^{(m)}(\omega)$ will depend only on the position μ of the atomic plane involved, so that for the F film $G_{ij}^{(m)}(\omega) = G_{F\mu}^{(m)}(\omega)$ and for the G monolayer $G_{ij}^{(m)}(\omega) = G_{G}^{(m)}(\omega)$.

After a two-dimensional Fourier transform to momentum space, one obtains L equations of motion for Green's functions of layer labeled by μ:

$$(\omega - A_{F1})G_{F1\mu}^{(m)}(\mathbf{q},\omega) + B_{F1}G_{F2\mu}^{(m)}(\mathbf{q},\omega) = A_{F1\mu}^{(m)}\delta_{1\mu}, \tag{8}$$

$$(\omega - A_{F\rho})G_{F\rho\mu}^{(m)}(\mathbf{q},\omega) + B_{F\rho}[G_{F(\rho+1)\mu}^{(m)}(\mathbf{q},\omega) + G_{F(\rho-1)\mu}^{(m)}(\mathbf{q},\omega)] = A_{F\rho\mu}^{(m)}\delta_{\rho\mu}, \tag{9}$$

$$\text{for} \quad \rho = 2,\dots,L-1,$$

$$(\omega - A_{FL})G_{FL\mu}^{(m)}(\mathbf{q},\omega) + B_{FL}G_{F(L-1)\mu}^{(m)}(\mathbf{q},\omega) = A_{FL\mu}^{(m)}\delta_{L\mu}, \tag{10}$$

where $\mathbf{q}$ is a wave vector in the planes parallel to the film surface and ν (or μ) = 1, ..., L. $A_{F\mu}$ and $B_{F\mu}$ for s.c. film F with spin $S_F = 2$ are given as follows:

$$A_{F1} = J_F\langle S_{F1}^z\rangle(4-\gamma_{\mathbf{q}}) + K_F\langle S_{F1}^z\rangle[2+\langle (S_{F1}^z)^2\rangle]/4 + J_F\langle S_{F2}^z\rangle + D\langle S_G^z\rangle, \tag{11}$$

$$A_{F\rho} = J_F\langle S_{F1}^z\rangle(4-\gamma_{\mathbf{q}}) + K_F\langle S_{F\rho}^z\rangle[2+\langle (S_{F\rho}^z)^2\rangle]/4 +$$
$$+J_F[\langle S_{F(\rho-1)}^z\rangle + \langle G_{F(\rho+1)}^z\rangle], \quad \text{for} \quad \rho = 2,\dots,L-1, \tag{12}$$

$$B_{F\mu} = J_F\langle S_{F\mu}^z\rangle \tag{13}$$

and $\gamma_{\mathbf{q}} = 2[\cos(q_x)+\cos(q_y)]$, $\langle S_{F\mu}^z\rangle = \langle S_{F(L+1-\mu)}^z\rangle$, $A_{F\mu} = A_{F(L+1-\mu)}$, $B_{F\mu} = B_{F(L+1-\mu)}$.

Equations (8), (9) and (10) can be written in the matrix form as

$$(\omega\mathbf{1} - \mathbf{P}_L)\mathbf{G}_{F\mu}^{(m)} = \mathbf{A}_{L\mu}^{(m)}, \quad \mu = 1,\dots L, \tag{14}$$

where $\mathbf{1}$ is the $L\times L$ unit matrix, $\mathbf{P}_L$ is the matrix of the set equations (8), (9) and (10), $\mathbf{G}_{F\mu}^{(m)}(\mathbf{q},\omega)$ is the Green's function vector $\begin{pmatrix} G_{F1\mu}^{(m)}(\mathbf{q},\omega) \\ \vdots \\ G_{FL\mu}^{(m)}(\mathbf{q},\omega) \end{pmatrix}$, and $\mathbf{A}_{F\mu}^{(m)}$ is the inhomogeneity vector with components

$$A_{F\mu\mu}^{(m)} = 2\left\langle (S_{F\mu\mu}^z - 1)^m S_{F\mu\mu}^z \right\rangle + \left\langle [(S_{F\mu\mu}^z-1)^m - (S_{F\mu\mu}^z)^m)][6 - S_{F\mu\mu}^z - (S_{F\mu\mu}^z)^2] \right\rangle. \tag{15}$$

In order to determine the correlation vector $\mathbf{C}_{F\mu}^{(m)} = \begin{pmatrix} C_{F1\mu}^{(m)}\delta_{1\mu} \\ \vdots \\ C_{FL\mu}^{(m)}\delta_{L\mu} \end{pmatrix}$ with correlation functions for each monolayer

$$C_{F\mu\mu}^{(m)} = \left\langle (S_{F\mu\mu}^z)^m S_{F\mu\mu}^- S_{F\mu\mu}^+ \right\rangle = 6\left\langle (S_{F\mu\mu}^z)^m \right\rangle - \left\langle (S_{F\mu\mu}^z)^{m+1} \right\rangle - \left\langle (S_{F\mu\mu}^z)^{m+2} \right\rangle, \tag{16}$$

we apply EVM. One sarts by diagonalizing the matrix $\mathbf{P}_L$ in (14)

$$\mathbf{L}P_L\mathbf{R} = \mathbf{\Omega} \equiv \begin{pmatrix} \omega_{F1} & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & \dots & \omega_{FL} \end{pmatrix}, \tag{17}$$

where $\mathbf{R}$ is matrix whose columns are the right eigenvectors of ion matrix $\mathbf{P}_L$, its inverse $\mathbf{L} = \mathbf{R}^{-1}$ contains the left eigenvectors as rows, where $\mathbf{L}R = \mathbf{R}L = \mathbf{1}$ and $\mathbf{\Omega}$ is a diagonal matrix with eigenvalues $\omega_{F\mu}$ of matrix $\mathbf{P}_L$.

Multiplying the equation of motion (14) from the left by $\mathbf{L}$ and inserting $\mathbf{1} = \mathbf{RL}$ yields

$$(\omega\mathbf{1} - \mathbf{\Omega})G^{(m)}_{F\mu} = A^{(m)}_{F\mu} \tag{18}$$

where we have introduced new vectors of Green's functions $G^{(m)}_{F\mu} = \mathbf{L}G^{(m)}_{F\mu}(\mathbf{q},\omega)$ and $A^{(m)}_{F\mu} = \mathbf{L}A^{(m)}_{F\mu}$. Each component $\mu = 1,\dots L$ of $G^{(m)}_{F\mu}$ has only a single pole

$$\mathcal{G}^{(m)}_{F\mu\mu} = \frac{\mathcal{A}^{(m)}_{F\mu\mu}}{\omega - \omega_{F\mu}}. \tag{19}$$

Now we can apply the spectral theorem to each component of the Green's function vector separately. For the μ component $\mathcal{C}^{(m)}_{F\mu\mu}(\mathbf{q})$ of the correlation vector in momentum space $C^{(m)}_{F\mu}(\mathbf{q}) = \begin{pmatrix} \mathcal{C}^{(m)}_{F1\mu}(\mathbf{q})\delta_{1\mu} \\ \vdots \\ C^{(m)}_{FL\mu}(\mathbf{q})\delta_{L\mu} \end{pmatrix}$ we obtain

$$\mathcal{C}^{(m)}_{F\mu\mu}(\mathbf{q}) = \frac{\mathcal{A}^{(m)}_{F\mu\mu}}{e^{\beta\omega_{F\mu}} - 1}, \tag{20}$$

where $\beta = 1/kT$. The correlation vector $C^{(m)}_{F\mu}(\mathbf{q})$ for the layer μ
can be written in the following form

$$C^{(m)}_{F\mu}(\mathbf{q}) = EA^{(m)}_{F\mu}, \tag{21}$$

where E is a diagonal matrix with elements $\mathcal{E}_{\mu\nu} = \delta_{\mu\nu}(e^{\beta\omega_{F\nu}} - 1)^{-1}$. Equation (21) can be rewritten as

$$\mathbf{L}C^{(m)}_{F\mu}(\mathbf{q}) = E\mathbf{L}A^{(m)}_{F\mu}. \tag{22}$$

The desired correlation vector $\mathbf{C}^{(m)}_{F\mu}$ is now obtained by multiplying (22) from the left by $\mathbf{R}$:

$$\mathbf{C}^{(m)}_{F\mu}(\mathbf{q}) = \mathbf{R}E\mathbf{L}A^{(m)}_{F\mu}. \tag{23}$$

Equation (23) is in momentum space. The complete system of integral equations obtained by Fourier transformation to coordinate space

$$\mathbf{C}^{(m)}_{F\mu} = \int d\mathbf{q}\mathbf{R}E\mathbf{L}A^{(m)}_{F\mu}. \tag{24}$$

Integral equations (24) have to be solved self-consistently. The momentum integral goes over $\mathbf{q}$ in the first Brillouine zone. From (24) for the correlation function $C^{(m)}_{F\mu\mu}$ we have

$$C^{(m)}_{F\mu\mu} = A^{(m)}_{F\mu\mu}\Phi_{F\mu\mu}, \tag{25}$$

where

$$\Phi_{F\mu\mu} = \frac{1}{\pi^2}\int_0^{\pi}\int_0^{\pi} dq_x d_y \sum_{\nu=1}^{L}\sum_{\kappa=1}^{L} R_{\mu\nu}\mathcal{E}_{\nu\kappa}\delta_{\nu\kappa}L_{\kappa\mu} \tag{26}$$

Here we have L correlation functions corresponding to the L-dimensional $\mathbf{P}_L$ matrix with L eigenvalues.

To elucidate the equation (25) we derive explicit expressions for $L = 3$ and spin $S = 2$. We need the inhomogeneities (15) and the correlations functions (16) with $m = 0$ and 1. Utilizing the relation $\langle\prod_{r=-S}^{r=S}(S_\mu^z - r\rangle = 0$, we find from (25) the spontaneous magnetization $\langle S_{F\mu}^z\rangle_{S=2}$ and the second moment $\langle(S_{F\mu}^z)^2\rangle_{S=2}$ for the monolayers 1 and 2:

$$\langle S_{F\mu}^z\rangle_{S=2} = \frac{2 + 9\Phi_{F\mu\mu} + 15\Phi_{F\mu\mu}^2 + 10\Phi_{F\mu\mu}^3}{1 + 5\Phi_{F\mu\mu} + 10\Phi_{F\mu\mu}^2 + 10\Phi_{F\mu\mu}^3 + 5\Phi_{F\mu\mu}^4}, \tag{27}$$

$$\langle(S_{F\mu}^z)^2\rangle = \frac{4 + 17\Phi_{F\mu\mu} + 27\Phi_{F\mu\mu}^2 + 20\Phi_{F\mu\mu}^3 + 10\Phi_{F\mu\mu}^4}{1 + 5\Phi_{F\mu\mu} + 10\Phi_{F\mu\mu}^2 + 10\Phi_{F\mu\mu}^3 + 5\Phi_{F\mu\mu}^4}. \tag{28}$$

For the spin $S = 6$ we obtain from (25), with $m \le 2S - 1 = 11$, for the spontaneous magnetization $\langle S_G^z\rangle_{S=6}$ of the monolayer G the following form

$$\langle S_G^z\rangle_{S=6} = \frac{(6 - \Phi_G)(1 + \Phi_G)^{13} + (14 + \Phi_G)\Phi_G^{13}}{(1 + \Phi_G)^{13} - \Phi_G^{13}} \tag{29}$$

where

$$\Phi_G = \frac{1}{\pi^2}\int_0^{\pi}\int_0^{\pi} \frac{1}{\exp(\beta\omega_G) - 1} dq_x dq_y \tag{30}$$

and

$$\omega_G = J_G\,\langle S_G^z\rangle\,(4 - \gamma_{\mathbf{q}}) + K_G\langle S_G^z\rangle(30 + \langle(S_G^z)^2\rangle)/36 + 2D\langle S_{F1}^z\rangle. \tag{31}$$

The formula for $\langle(S_G^z)^2\rangle$ is too large, hence it is omitted.

Magnetizations $\langle S_{F\mu}^z\rangle$ ($\mu = 1,2, \ldots, L$) and $\langle S_G^z\rangle$ simultaneously approach zero at the Curie temperature T_C, ($T_C^{F\mu} = T_C^G \equiv T_C$), but all magnetizations do not tend to zero at T_C with the same degree of convergence. Therefore, it is convenient to scale all the quantities in terms of $\langle S_G^z\rangle$, which does not vanish even in the immediate vicinity of T_C.

In the high-temperature approximation as $T \to T_C$ we have

$$e^{\beta\omega_{F\mu}} - 1 \cong \langle S_G^z\rangle J_F\beta_C\widetilde{\omega}_{F\mu}, \qquad e^{\beta\omega_G} - 1 \cong \langle S_G^z\rangle J_G\beta_C\widetilde{\omega}_G, \tag{32}$$

where $\widetilde{\omega}_{F\mu} = \omega_{F\mu}/J_F\langle S_G^z\rangle$ and $\widetilde{\omega}_G = \omega_G/J_G\langle S_G^z\rangle$. Parameters Φ_G and $\Phi_{F\mu\mu}$ in this case can be approximated by

$$\Phi_{F\mu\mu} \cong \frac{1}{\langle S_G^z\rangle J_F\beta_C}\widetilde{\Phi}_{F\mu\mu}, \qquad \Phi_G \cong \frac{1}{\langle S_G^z\rangle J_G\beta_C}\widetilde{\Phi}_G, \tag{33}$$

the tilde symbol designates a scaled quantity in terms of $\langle S_G^z\rangle$. Then from (28) at high-temperature ($\Phi_G \gg 1$) and (33) we obtain

$$\frac{kT_C}{J_G} \cong \frac{1}{14\widetilde{\Phi}_G}. \tag{34}$$

We have to solve $L+1$ equations self-consistently for $L+1$ unknowns: critical temperature (34) and relative magnetizations

$$\frac{\langle S^z_{F\mu}\rangle}{\langle S^z_G\rangle} = \frac{28J_F}{J_G}\frac{\widetilde{\Phi}_G}{\widetilde{\Phi}_{F\mu\mu}}. \tag{35}$$

3 Results and Discussion

Solving the self-consistent equations (34) and (35) by performing numerical integrations, we can estimate the magnetization of each atomic plane and the critical temperature. Of special interest is how to describe the interaction of magnitude D at the interface between the the film F with exchange interaction J_F and the monolayer G with exchange interaction J_G and the single-ion anisotropy in both films would influence the value of the layer magnetizations and the critical temperature of the whole system. A similar problem has been solved in [6] where only the effect of the coupled interaction in the interface between the layers of two-element multilayer film on the layer magnetization and critical temperature has been studied.

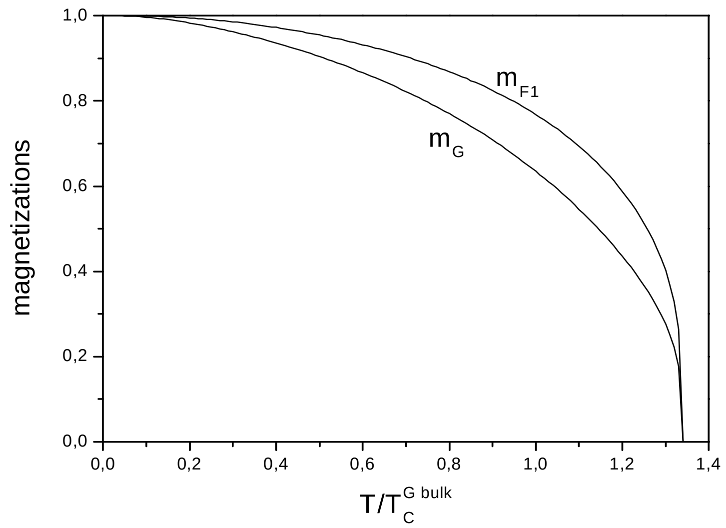

Fig. 1 Magnetization of the first atomic plane in the ferromagnet F $m_{F1} = \langle S^z_{F1}\rangle / S_F$ and the magnetization of the ferromagnet G $m_G = \langle S^z_G\rangle / S_G$ in the case $D/J_G = 2, \eta = J_F/J_G = 8, K_F/J_F = 0.1$ and $K_G/J_G = 0$.

In Fig. 1 we show the relative magnetization of each atomic plane $m_{F\mu} = \langle S_{F\mu}\rangle/S_F$ and $m_G = \langle S_G\rangle/S_G$ as a function of the reduced temperature T/T_C^{Gbulk} (we assume the bulk Curie temperaure $T_C^{Gbulk} = 200$ K of the ferromagnet G) for case $L = 4$, $D/J_G = 2, \eta = J_F/J_G = 8$, $K_F/J_F = 0.1$ and $K_G/J_G = 0$. Applying the model of the simple cubic lattice with $S_G = 6$, the effective exchange parameter J_G of the ferromagnet G is determined from the equation, $T_C^{Gbulk} = \frac{2S_G(S_G+1)}{C}\frac{J_G}{k}$, where k is Boltzmann's constant and C is a constant (for the simple cubic lattice $C = 1.516386 : \frac{J_G}{k} = 3.6$). From Fig. 1, we can see that the magnetizations m_G and m_{F1} (temperature dependence of magnetization of layer 2 is very close to m_{F1}) simultaneously approaches zero at the reduced Curie temperature T_C/T_C^{Gbulk} of the whole system. In two-element multilayer films there is one well defined critical temperature.

Next, we will consider the phase diagrams in $(D/J_G, T_C/T_C^{Gbulk})$ and $(J_F/J_G, T_C/T_C^{Gbulk})$ planes. Fig. 2 illustrates the $(D/J_G, T_C/T_C^{Gbulk})$ phase diagram for different number of the thin film F indicated by the numbers and for the case when the single-ion anisotropy parameters in the film F and monolayer G are $K_F/J_G = 0.1$ and $K_G/J_G = 0$, respectively, and when exchange parameter in the film F has the value $J_F = 8J_G$. According to these results, the critical parameter $D_C/J_G = 2.195$ can be defined as that particular value D/J_G at which the Curie temperature of the two-element multilayer film does not depend on the thickness of the film F (the cross-over point). The cross-over point in thin films defines the critical temperature T_C^{bulk} of the three-dimensional bulk system, where the surface is of no importance. We note that the bulk critical temperature for the ferromagnet F is $T_C^{Fbulk} = \frac{2S_F(S_F+1)}{C}\frac{J_F}{k} = 228$ K and the critical temperature of the whole system is $T_C = 1.322 \times 200 = 290.8$ K. The critical temperature of the whole system is greater than T_C^{Fbulk} because a spontaneous magnetization results from strong F-G pair interaction in the interface, the free energy of the whole system goes down, so that the ferromagnetic state is stabilized at the high temperature.

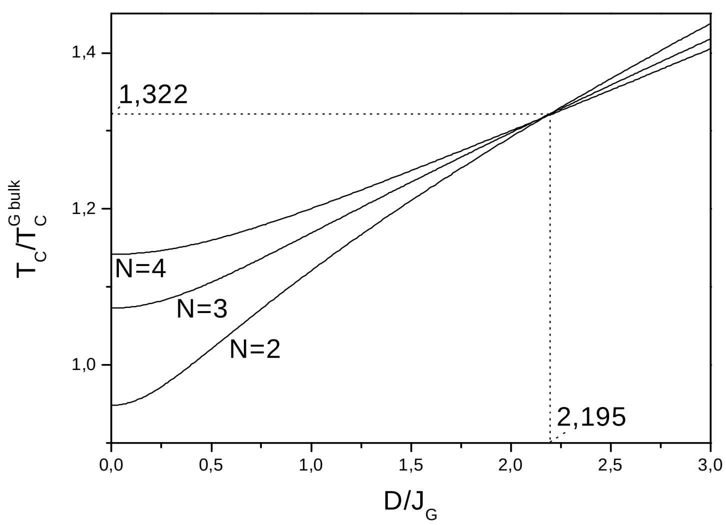

Fig. 2 The reduced critical temperature T_C/T_C^{Gbulk} of the multilayer system as a function of uniaxial anisotropy parameter D/J_G at the interface with differnet values of thickness of F film labeled by number and in the case when $K_F/J_F = 0.1, K_G/J_G = 0$, and $\eta = J_F/J_G = 8$.

In Fig. 3 is plotted the $(J_F/J_G, T_C/T_C^{Gbulk})$ phase diagram for different number of the F layers indicated by the numbers and for the case when the single-ion anisotropy parameters in F layers and G monolayer are $K_F/J_F = 0.1$ and $K_G/J_G = 0$, respectively, and when the interaction parameter in the interface $D/J_G = 2.195$. The results correspond with the results in Fig. 2. In this case we observe the second cross-over point $\eta_C = 8$ at which the Curie temperature of the F thin film (and also of the whole system) does not depend on the thickness of the thin film F.

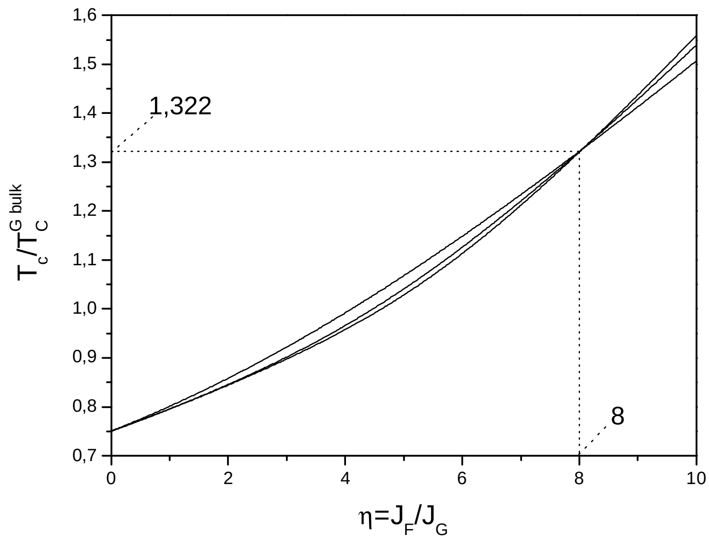

Fig. 3 The reduced critical temperature T_C/T_C^{Gbulk} of the multilayer system as function of $\eta = J_F/J_G$ with different values of thickness of the F film labeled by number and in the case when the uniaxial exchange anisotropy parameter $D/J_G = 2.195$, the parameter of the single-ion anisotropy in the F film is $K_F/J_F = 0.1$ and in the G monolayer is $K_G/J_G = 0$.

Finally, we present in Figs. 4 and 5 the location of the cross-over points D_C/J_G and η_C as a function of the single-ion anisotropy in F film and G monolayer. From Fig. 4 we see that when K_F/J_F is fixed (in the case $\eta = 8$), the critical lines D_C/J_G decreases smoothly from maximum value at $K_G/J_G = 0$ to $K_G/J_G = 0.1$. On other hand, when K_G/J_G is fixed, we observe that the critical line increases with increasing K_F/J_F from the minimum value at $K_F/J_F = 0$. In the case $J_F > J_G$, the critical temperature of the whole sandwich system for $D > D_C/J_G$ is greater than the bulk critical temperature of the ferromagnets F and G.

From Fig. 5 we see that when K_F/J_F is fixed (in the case $D_C/J_G = 2$), the critical lines η_C increases smoothly from minimum value at $K_/J_G = 0$ to $K_G/J_G = 0.1$. On other hand, when K_G/J_G is fixed, we observe that the critical line decreases with increasing K_F/J_F from maximum value at $K_F/J_F = 0$. The behavior of the cross-over point η_C at the fixed value of K_F/J_F or K_G/J_G has opposite tendency relative to the cross-over point D_C/J_G. In other words, the influence of the single-ion anisotropy in the F film and in the G monolayer on the cross-over points D_C/J_G and η_C is different. In the case $D/J_G = 2$, the critical temperature of the whole sandwich system for $\eta > \eta_C$, is greater than the bulk critical temperature of the ferromagnets F and G.

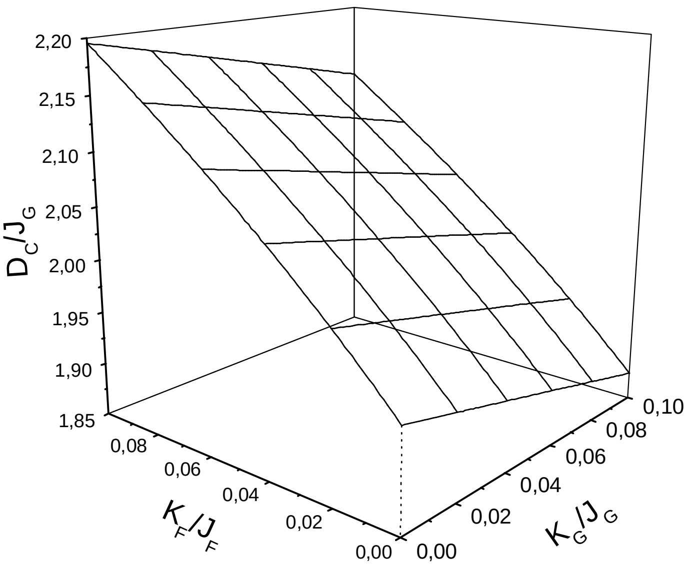

Fig. 4 Critical ratio D_C/J_G as a function of the single-ion anisotropy in the F film and in the G monolayer, in the case $\eta = 8.$. Critical plane locates the cross-over points.

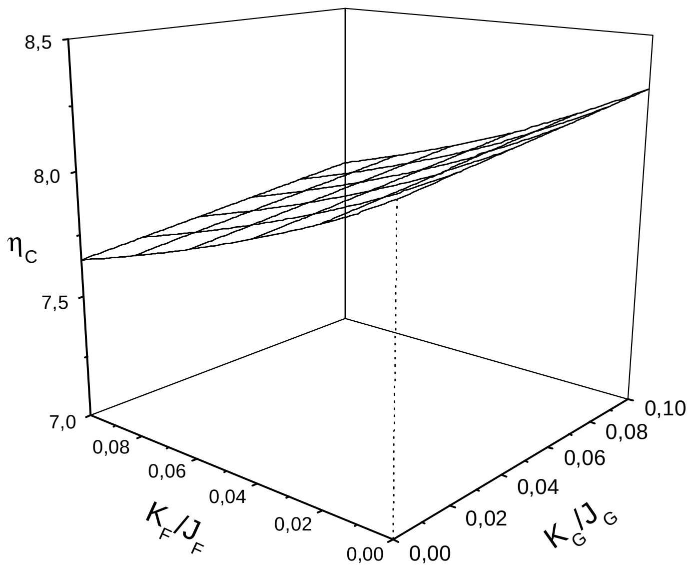

Fig. 5 Critical parameter η_C as a function of the single-ion anisotropy in the F film and in the G monolayer, in the case $D/J_G = 2$. Critical lines locate the cross-over points.

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