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Natural brane-confinement from massive Z_2-spontaneously broken Kaluza–Klein excitations in the bulk

Research Article

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Abstract: For a real scalar field minimally coupled to bulk gravity, in five dimensions, we analytically solve the Gordon equation, near one of the degenerated vacua of an effective potential with a spontaneously broken Z_2-symmetry. Dealing with the back-reaction from the excited massive modes on the whole scale function, we are pointing out that the lighter excitations of the scalar in the bulk turn more and more the warp function into the one of a partition on the confined brane.

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1. Introduction

The assumption that our Universe may be a four-dimensional hypersurface embedded in a background spacetime of higher dimension has a long history, which can be traced back to the pioneering work of Kaluza and Klein [1, 2], where extra dimensions are compacted up to an unobservable size. More recently, the model bearing the names of Randall and Sundrum (RS) [3], where a single brane with non-vanishing tension is embedded in five-dimensional anti-de Sitter spacetime and the bulk gravity is strongly coupled to the dynamics on the brane, had attracted an enormous attention, mainly because these extra dimensions might be much larger than the Planck scale [4], leading to measurable effects [5]. The use of an extra dimension has been considered essential once it has been shown that the strong curvature region of the solutions are cutting off its size giving rise to macroscopic 4D gravity without a cosmological constant [6]. Even though the cosmological constant Λ was non-zero, a significant cancellation of the effective cosmological constant has been signaled in [7], where domain walls embedded in curved backgrounds have been considered as an approximation for braneworld scenarios. While dealing with the stability of the geodesic motion of a test particle on a hypersurface in 5-dimensional spacetimes [8], it has been shown that, in the RS type II model, the matter is practically expelled from the brane [9], and additional mechanisms of confine-

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ment are needed [10]. In order to avoid the curvature singularity at the location of a thin wall, gravity has to be coupled to a scalar field, and both the warp factor and the scalar wave function have been parametrized in terms of a model superpotential [11, 12]. However, when going from the thick walls with four-dimensional Minkowski slices to bent solutions, one needs to solve directly the non-linear equations of motion. In this respect, fluctuations of the gravitational field around the solutions have been studied in order to describe how bulk modes interact with matter on the domain walls with de Sitter and anti-de Sitter geometries [13]. Concerning the Gordon equation, whose excited mode-solutions are derived in the present work, for a potential with a spontaneously broken symmetry, this has been intensively studied in different approaches as, for example, the case of the Schwarzschild–de Sitter solution embedded in a Randall–Sundrum brane model, pointing out that the structure of the fifth dimension affects the nature of the Hawking radiation observed in four-dimensional spacetime [14]. Also, the equations describing a spinning test object moving in a circular orbit in a Schwarzschild-like spacetime augmented by one extra dimension have been exactly solved, suggesting that the spin axis precesses at different rate in four and five dimensions and this fact could be used to test the dimensionality of the world, using a gyroscope in Earth orbit [15]. Recently, the scalar perturbations in a Randall–Sundrum-type model of inflation have been quantized, dealing with the inflaton field confined to a single brane embedded in a five dimensional anti-de Sitter spacetime. In contrast to the standard four-dimensional theory, it has been shown that the gravitational back-reaction can strongly affect the behavior of inflaton perturbations on sub-horizon scales [16].

2. Scalar fields in the bulk

Let us start with the 5-dimensional space, described by the metric

$$ds_5^2 = e^{2f(w)}\eta_{ik}\,dx^i dx^k + (dw)^2\,, \tag{1}$$

with $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = t$, $x^5 = w$, where η_{ik} is the usual Minkowski metric and the warp factor depends only on the extra dimension, $x^5 = w$. As in four dimensions, we are introducing the pseudo-orthonormal frame $\{e_a\}_{a=\overline{1,5}}$, so that $ds_5^2 = \eta_{ab}\omega^a\omega^b$, with $(\eta_{ab}) = \text{diag}[1,1,1,-1,1]$, whose corresponding dual base is

$$\omega^i \;=\; e^{f(w)}\,dx^i\;,\quad \omega^5 \;=\; dw\,, \tag{2}$$

with $i = \overline{1,4}$. Using the Cartan structure equations, we derive the essential connection coefficients

$$\Gamma_{\alpha 5\alpha} = f_{,5}\;,\quad \Gamma_{454} = -f_{,5}\;,\quad \alpha = \overline{1,3}\,, \tag{3}$$

and the respective components of the curvature tensor,

$$\begin{aligned} R_{\alpha\beta\alpha\beta} &= -R_{\alpha4\alpha4} = -(f_{,5})^2\;, \\ R_{\alpha5\alpha5} &= -R_{4545} = -\left[f_{,55} + (f_{,5})^2\right]. \end{aligned} \tag{4}$$

Thus, the non-vanishing Ricci and Einstein tensor components are given by

$$\begin{aligned} R_{\alpha\alpha} &= -R_{44} = -\left[f_{,55} + 4(f_{,5})^2\right]; \\ R_{55} &= -4\left[f_{,55} + (f_{,5})^2\right], \end{aligned} \tag{5}$$

and

$$\begin{aligned} G_{\alpha\alpha} &= -G_{44} = 3\left[f_{,55} + 2(f_{,5})^2\right]; \\ G_{55} &= 6(f_{,5})^2\;, \end{aligned} \tag{6}$$

so that the scalar curvature reads

$$R = -4\left[2f_{,55} + 5(f_{,5})^2\right]. \tag{7}$$

In the spacetime endowed with the metric (1), the real scalar field minimally coupled to bulk gravity is described by the following Lagrangian,

$$\mathcal{L}[\phi] = -\frac{1}{2}\eta^{ab}\phi_{|a}\phi_{|b} - V(\phi)\,. \tag{8}$$

The Einstein field equations, $G_{ab} = kT_{ab}$, with the energy–momentum tensor

$$T_{ab}[\phi] = \phi_{|a}\phi_{|b} - \frac{1}{2}\eta_{ab}\left[\eta^{cd}\phi_{|c}\phi_{|d} + 2V(\phi)\right], \tag{9}$$

have the explicit form

$$\begin{aligned} f_{,55} + 2(f_{,5})^2 &= -\frac{k}{6}\left[(\phi_{,5})^2 + 2V(\phi)\right], \\ 2(f_{,5})^2 &= \frac{k}{6}\left[(\phi_{,5})^2 - 2V(\phi)\right], \end{aligned} \tag{10}$$

while the 5D Gordon-type equation reads

$$\begin{aligned} &\phi_{,55} + 4f_{,5}\,\phi_{,5} + e^{-2f}\left[\Delta - \frac{\partial^2}{\partial t^2}\right]\phi = \frac{dV}{d\phi}; \\ &\text{with} \qquad \Delta = \delta^{\alpha\beta}\,\partial^2_{\alpha\beta}\,. \end{aligned} \tag{11}$$

By taking the sum and the difference of the two relations in (10) and considering ϕ as a function of the extra dimension alone, we get the following coupled differential equations:

$$
\begin{aligned}
&(a)\quad \frac{d^2 f}{dw^2} + 4\left(\frac{df}{dw}\right)^2 = -\frac{2k}{3}\, V(\phi)\,,\\
&(b)\quad \frac{d^2 f}{dw^2} = -\frac{k}{3}\left(\frac{d\phi}{dw}\right)^2,\\
&(c)\quad \frac{d^2\phi}{dw^2} + 4\,\frac{df}{dw}\frac{d\phi}{dw} = \frac{dV}{d\phi}\,.
\end{aligned}
\tag{12}
$$

Even though the above system of equations for gravity and scalar fields looks seemingly simple and has been numerically studied, for different forms of the potential [11–13], its analytic treatment has not been dealt with satisfactorily. While dealing with exactly solvable models with integrable bulk potential, the most common method, inspired from gauged supergravity, is to express both the warp factor and the scalar wave function in terms of a model superpotential [17].

Let us start with the following particular solutions of the system (12):

$$f(w) = -\frac{kC^2}{12}\,\ln\left[\cosh(aw)\right], \tag{13}$$

and

$$\phi(w) = C \arctan\left[\tanh\left(\frac{aw}{2}\right)\right], \tag{14}$$

for which the potential, in terms of $\phi(w)$, is:

$$V(\phi) = -\frac{ka^2C^4}{24} + \left(1 + \frac{kC^2}{3}\right)\frac{a^2C^2}{8}\cos^2\left(\frac{2\phi}{C}\right). \tag{15}$$

At a first sight, one is tempted to put $kC^2 = -3$, so that the above expression simplifies drastically to a constant,

$$V_0 = -\frac{3a^2}{8k}\,,$$

and the solution (13) is the hyperbolic metric

$$f(w) = \frac{1}{4}\ln\left[\cosh\left(aw\right)\right], \tag{16}$$

which has been used as a *trapping solution* for explaining the matter-confinement [18]. However, in this assumption, the scalar field wave function (14) is imaginary, vanishing once we come to the brane, at $w = 0$, but still physically unacceptable.

Next, in order to find the particle content of this model, one has to analyze the field small oscillations around the extremum of the effective potential (15). As it can be noticed, the series expansion of the cosine function leads to the familiar form

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4\,, \tag{17}$$

with two degenerate minima,

$$V_0 \equiv V(\phi_\pm) = -\mu^4/(4\lambda)\,. \tag{18}$$

For $\mu^2 = \mu_5^2 > 0$, the zero KK Higgs mode gets a non-vanishing vacuum expectation value and turns from a massless into a massive degree of freedom, as in the usual 4D Higgs mechanism [9].

With the zero-mode solution (16), let us move to the excited states near one of the vacua, $\phi = \phi_- + \varphi$ and keep only the mass term in the potential (17) written as $V(\varphi) = \mu^2\varphi^2 + \ldots$ Consequently, the Gordon Eq. (12)c reads

$$
\begin{aligned}
&\frac{d^2\varphi}{dw^2} + a\tanh(aw)\frac{d\varphi}{dw} - 2\mu^2\varphi = 0\,,\\
&a \equiv \mu^2\sqrt{\frac{2k}{3\lambda}}\,.
\end{aligned}
\tag{19}
$$

With the new integration variable $\zeta = (1 - i\sinh(aw))/2$, this finally becomes

$$
\begin{aligned}
&\zeta(1-\zeta)\frac{d^2\varphi}{d\zeta^2} + (1-2\zeta)\frac{d\varphi}{d\zeta} + \varepsilon^2\varphi = 0;\\
&\varepsilon^2 \equiv \frac{2\mu^2}{a^2}
\end{aligned}
\tag{20}
$$

and its solutions are the hypergeometric functions ${}_2F_1[\alpha, \beta, 1;\, \zeta]$, with

$$
\begin{aligned}
\alpha &= -\frac{1}{2}\left[\sqrt{4\varepsilon^2+1} - 1\right];\\
\beta &= +\frac{1}{2}\left[\sqrt{4\varepsilon^2+1} + 1\right].
\end{aligned}
$$

Since α seems already negative, we impose on it the quantization condition $\alpha = -n$, with $n \in \mathbb{N}$, yielding

$$\beta = n + 1\,, \qquad \varepsilon^2 = n(n+1)\,. \tag{21}$$

Hence, the massive excitations will be described by the modes

$$\varphi_n(w) \sim F\left\{-n,\, n+1,\, 1\,;\, \frac{1}{2}\left[1 \pm i\sinh(aw)\right]\right\}. \tag{22}$$

These can be expressed in terms of Legendre polynomials as

$$\varphi_\ell(w) = \mathcal{N}_\ell P_\ell \left[\pm i \sinh(aw)\right], \tag{23}$$

where the integration constant $\mathcal{N}_\ell$ is respectively real or imaginary, for ℓ even or odd, so that the solution is always a real function of w.

With respect to the boundary conditions we have used, let us notice the following: Starting with the usual eigenvalue problem for the L^2−operator, one can consider only the $m = 0$−states, so that the equation reads:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dY_\ell^0}{d\theta}\right) + \ell(\ell+1)Y_\ell^0 = 0 .$$

Shifting the variable θ to $\psi = \theta - \pi/2$, the equation becomes

$$\frac{d^2 Y_\ell^0}{d\psi^2} - \tan\psi \frac{dY_\ell^0}{d\psi} + \ell(\ell+1)Y_\ell^0 = 0$$

and therefore, applying here the Wick rotation $\psi = i(aw)$, it gets into the form

$$\frac{d^2 Y_\ell^0}{dw^2} + a \tanh(aw)\frac{dY_\ell^0}{dw} - \ell(\ell+1)a^2 Y_\ell^0 = 0,$$

which is precisely the same as (19), with the mass parameter quantization law $2\mu^2 = \ell(\ell+1)a^2$. Thence, the regular and univocally-defined solutions

$$Y_\ell^0(\theta) = \sqrt{\frac{2\ell+1}{4\pi}}\, P_\ell(\cos\theta)$$

go by this complex extension, with $\cos\theta = -i\sinh(aw)$, into the eigenmode solutions (23) of the Higgs field excitations.

Using (21), one may notice that, unlike the normal tower of Kaluza–Klein spectrum, which is increasing with the quantum number, now, the mass appearing from the Z_2-spontaneous violation in the bulk got quantized in a non-trivial manner:

$$m_0^2 = 2\mu^2 \quad \text{and} \quad 2\mu^2 = n(n+1)a^2 \tag{24}$$

leading to the law

$$m_n = \left[\frac{6\lambda/k}{n(n+1)}\right]^{1/2} . \tag{25}$$

It basically comes from the fact that the scalar field in the bulk and its induced 5D-gravity should share a common vacuum state

$$-\frac{3a^2}{8k} = V_0 = -\frac{\mu^4}{4\lambda} ,$$

as an exact solution to the Einstein–Gordon equations. Thus, considering the five dimensional Einstein constant k as universal and the quartic self-interaction coupling parameter λ as a constant of the model, it yields that the length parameter a^{-1} along the fifth dimension and the mass parameter μ of the scalar in the bulk are no longer independent and only the values

$$\mu_n = \left[\frac{3\lambda/k}{n(n+1)}\right]^{1/2}$$

are allowed for the latter. In view of the confinement results derived in the next section, which is investigating the back reaction process, the mass quantization law (25) could probably explain why our Universe is mainly filled in with much lighter particles than the yet hypothetical ones living above the SUSY and up to the Planck scale.

At the end of this section, we would like to stress that, in the absence of the SSB, there is no ground state of negative potential $V_0 = -3a^2/(8k)$, so that the metric function (13) and the field (14), which must lead to the potential (15), would no longer be an exact solution to the Einstein–Gordon equations (12). Thence, a minimally coupled scalar field with no quartic interaction can be only considered as a test field satisfying along the fifth dimension the differential Eq. (19), but with m^2 instead of $2\mu^2$, so that the mass quantization relation would simply read $m_n^2 = n(n+1)a^2$, representing the usual KK tower, with a^{-1} being the characteristic length of the fifth dimension.

3. Back-reaction from the excited modes

We assume that the existence of the scalar field depending on the extra dimension might give rise to some new effects on the bulk, when the higher modes are excited.

In order to see the feedback of massive excitations (23) on the bulk scale function, we use the recurrence relation of Legendre polynomials with respect to the derivatives, [19], and we immediately obtain for the curvature Eq. (12)b the following two **R**-valued forms, for $\ell = 2n$ and $\ell = 2n+1$, respectively:

$$\frac{d^2 f_\ell}{dw^2} = -\frac{k}{3}\left|\mathcal{N}_{2n}\right|^2 \times \frac{(2n)^2 a^2}{\cosh^2(aw)} \left\{\sinh(aw) P_{2n}\left[\pm i \sinh(aw)\right] + i P_{2n-1}\left[\pm i \sinh(aw)\right]\right\}^2 ,$$
$$\frac{d^2 f_\ell}{dw^2} = -\frac{k}{3}\left|\mathcal{N}_{2n+1}\right|^2 \times \frac{(2n+1)^2 a^2}{\cosh^2(aw)} \left\{i \sinh(aw) P_{2n+1}\left[\pm i \sinh(aw)\right] - P_{2n}\left[\pm i \sinh(aw)\right]\right\}^2 . \quad (26)$$

The corrected scale function, $S = e^f$, namely

$$S(w) = S_{(0)}(w) \times e^{f_\ell(w)} ,$$
$$\text{where} \quad S_{(0)} = \exp\left\{\frac{1}{4}\ln\left[\cosh(aw)\right]\right\} , \quad (27)$$

is plotted in Fig. 1, for the even modes, $\ell \in \{2,4,6\}$ (Fig. 1a), and for the odd ones, $\ell \in \{1,3,5,7\}$ (Fig. 1b), the dotted curve representing the unperturbed scale function $S_{(0)}$. As it can be noticed, the even modes lead to scale functions of smaller amplitude in comparison to the one generated by the odd ones. Also, and more importantly, the region in the bulk where the scale function is getting actually constant turns more and more into a rectangular *tooth* resembling a sort of partition function for the brane. Actually, from a geometrodynamic perspective, it really stands for the brane in the sense that the lighter excitations of the scalar in the bulk turn more and more the warp function into the one of a partition on the confined brane.

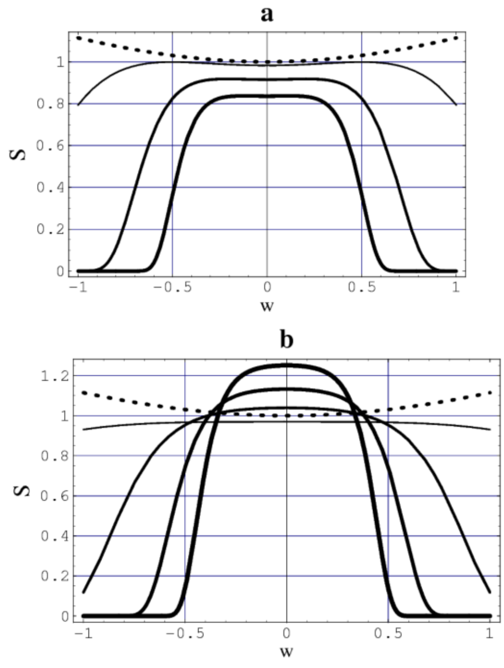

Figure 1. The total cosmological scale function (27), for the even modes, with $\ell = \{2,4,6\}$ (Fig. a) and for the odd modes, with $\ell = \{1,3,5,7\}$ (Fig. b), the dotted curve representing the unperturbed scale function $S_{(0)}$. As ℓ goes to larger values, the thickness of the plots is increasing.

4. Concluding remarks

In the five-dimensional spacetime generally described by the metric (1), we employ the usual Cartan formalism to derive the essential components of the Riemann, Ricci and Einstein tensors and the scalar curvature. For a real scalar field minimally coupled to bulk gravity, the system (12) is satisfied by the solutions (13) and (14) and the potential is given by (15). Although the hyperbolic metric (16) has been extensively used as a *trapping solution* of multidimensional Einstein equations, for explaining the matter-confinement [18], in agreement with the anzats of Rubakov–Shaposhnikov [20], the imaginary scalar wave function (14) is physically unacceptable. Thus, we turn to an effective potential with a spontaneously broken symmetry and solve the Gordon equation near one of the degenerated vacua. Our main motivation has come from the fact that five-dimensional extensions of the Standard Model based on the powerful mechanism of spontaneous symmetry breaking have been recently considered for generating masses for all the KK gauge modes, as in the usual Higgs mechanism [9]. Moreover, since KK states are now at the edge of accessibility at the LHC, Higgs fields hosted in the bulk require a better understanding of their dynamics. Finally, using the real solutions (23) for the odd and even modes, we study the back-reaction on the whole scale function, which is represented in Fig. 1, pointing out the brane-confinement in the bulk coming from the massive Z_2-spontaneously broken scalar excitations.

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