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Interfacial wave propagation due to an interfacial line source

Research Article

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Abstract: Interfacial wave propagation parallel to a dielectric interface has been studied by considering an electric current line source present at the interface. The first order asymptotic evaluation of field components shows a null of the electric field at the interface. An amplitude null represents an unstable structure in the phase map and a phase front discontinuity across the interface. Higher order asymptotic evaluation has been employed to gain further insight into this propagation problem. The results show that the wavefronts need not be discontinuous. The continuity of the phase fronts is preserved with the help of interesting and stable structures such as saddle points and center points in the phase map of the electric field in both half spaces.

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1. Introduction

Electromagnetic wave propagation at the interface of two different dielectric media is important in many applications. For example, in geo-physical problems, it helps to analyze sub-surface properties of various materials forming this configuration. This type of propagation has also proved helpful in determining the properties of leaky waves [1, 2]. The same is true for micro-strip structures [3].

The exact solution to the problem of electromagnetic wave propagation parallel to a dielectric interface is not available. As the phase velocity in the two dielectric media is different, it is not apparent in what way the velocity of the wave will change from one medium to the other. An asymptotic first-order solution to the problem has been given by Engheta and Papas [4]. They show that the radiation field has two important effects. One effect is surface field extinction, *i.e.*, the radiation pattern of the electric field has a null at the interface. The other effect is called sub-surface peaking, *i.e.*, the radiation pattern of the electric field displays a maximum at critical angle in the medium with higher permittivity. Volski and Vandenbosch [5], also found a radiation pattern due to a line source carrying electrical current placed on a dielectric slab. They employed the expansion wave concept and the geometrical theory of diffraction to solve their problem. Their asymptotic results also show null field at the interface. In both the problems discussed above, the dominant term in the asymptotic expansion of the electric field has a null at the interface. The exact field on the inter-

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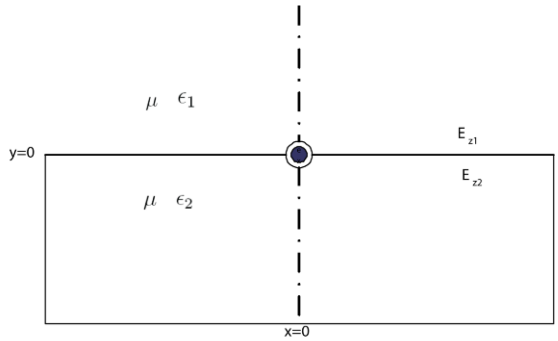

Figure 1. Geometry of the problem.

face for an interfacial line source is given by Engheta and Papas [6]. However, this exact expression is not valid in the vicinity of the interface. This leaves the fundamental question of phase velocity transition from one medium to another unanswered. Moreover, null field along the interface presents an structurally unstable Poynting vector field. Any slight perturbation can produce many different phase front maps or Poynting vector field configurations [7]. Therefore, it is desireable to find a structurally stable solution to the problem of phase velocity near the dielectric interface.

2. Formulation

The geometry of the problem is presented in Fig. 1. A dielectric with permittivity ϵ_1 fills the region $y > 0$ and another dielectric with permittivity ϵ_2 fills the region $y < 0$. The permeability of both half spaces is assumed to be μ. A linear source of electric current is placed coincident with the z-axis on the interface of the dielectric media. The current distribution associated with this source is given as

$$\mathbf{J} = I\delta(x)\delta(y)\exp(-i\omega t)\mathbf{a_z},$$

where $\delta(x)$, $\delta(y)$ are Dirac delta functions, I is the current and $\mathbf{a_z}$ is the unit vector along the z-axis.

The time harmonic dependence, $e^{-i\omega t}$, will be suppressed in further calculations. In this situation, Maxwell's equations dictate that only the z-component of the electric field, E_z, will be present,which satisfies the Helmholtz equation,

$$\nabla^2 E_z + k^2 E_z = -i\omega\mu I\delta(x)\delta(y), \tag{1}$$

where

$$k^2 = \begin{cases} \omega^2\mu\epsilon_1 = k_1^2, & y > 0; \\ \omega^2\mu\epsilon_2 = k_2^2 = n^2k_1^2, & y < 0. \end{cases}$$

The refractive index in the region $y < 0$ is $n = \sqrt{\epsilon_2/\epsilon_1}$. Let the electric field for $y > 0$ be given as E_{z1}, and for $y < 0$, as E_{z2}. The solution of (1) is given as

$$E_{z1} = \frac{\omega\mu I}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(ik_x x + ik_{1y}y)}{(k_{1y}+k_{2y})}dk_x, \tag{2}$$

$$E_{z2} = \frac{\omega\mu I}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(ik_x x - ik_{2y}y)}{(k_{1y}+k_{2y})}dk_x, \tag{3}$$

where

$$k_{1y} = \sqrt{k_1^2 - k_x^2},$$

and

$$k_{2y} = \sqrt{k_2^2 - k_x^2}.$$

The integrals above cannot be evaluated explicitly, except on the interface. On the interface, at $y = 0$, (2) and (3) are identical and the field is given as [6]

$$E_z = E_{z1} = E_{z2} = \frac{\omega\mu I}{2(n^2-1)k_1}\left\{\frac{1}{|x|}H_1^{(1)}(k_1|x|) - \frac{n}{|x|}H_1^{(1)}(nk_1|x|)\right\}. \tag{4}$$

Asymptotic expressions for the dominant terms of Eqs. (2) and (3) have been given by Engheta and Papas [4]. These expressions have the following form in cylindrical polar coordinates:

$$E_{z1} \sim \frac{A(\theta)}{\sqrt{k_1 r}}e^{ik_1 r}, \tag{5}$$

$$E_{z2} \sim \frac{B(\theta)}{\sqrt{k_2 r}}e^{ik_2 r}, \tag{6}$$

where $A(\theta)$ and $B(\theta)$ are radiation field patterns with a null at $\theta = 0$. The contours of constant phase are given in Fig. 2 for the solution given above. The phase and phase velocity are both ambiguous at the interface due to the null in the asymptotic expressions (5) and (6). There is an abrupt change in the phase velocity on both sides of the interface. This is only the expression for the dominant term. The exact field on the interface decays as $(k|x|)^{-3/2}$ according to Eq. (4). It has been shown in [7] that when there is a null in the dominant term of the electric field

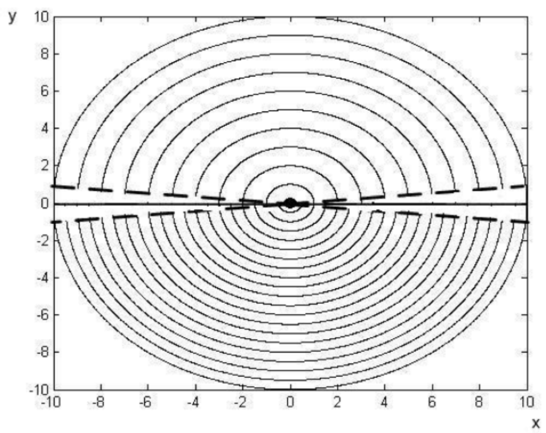

Figure 2. Circular phase fronts due to line source in the two media.

along a continuous curve, the addition of small terms to the field expression given by the dominant term will completely change the phase velocity picture. Therefore, to investigate the behavior of the phase velocity near the interface, higher order asymptotic terms which decay as $(kr)^{-3/2}$ will have to be taken into account.

3. Asymptotic solution

An asymptotic evaluation of the field above and below the interface will be carried out to find the higher order terms. Since the field is symmetric in x, it will be considered only for $x > 0$. The field expression for E_{z1} is an integral which contains branch point singularities at $k_x = \pm k_1$ and $k_x = \pm k_2$. After making the following transformations,

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad k_x = k_1\cos\alpha = k_1\cos(\eta + i\zeta)$$

in the range $0 < \theta < \pi/2$, the integral in (2) is now represented as

$$E_{z1} = -\frac{\omega\mu I}{2\pi}\int_P f(\alpha)\exp\{ik_1 r\cos(\alpha-\theta)\}\,d\alpha, \qquad (7)$$

where

$$f(\alpha) = \frac{\sin\alpha}{(\sin\alpha + \sqrt{n^2 - \cos^2\alpha})}.$$

The path of integration P runs as shown in Fig. 3. A first order saddle point at $\alpha = \theta$, a pair of branch points at $i\cosh^{-1}(n)$ and $\pi - i\cosh^{-1}(n)$ are also shown in Fig. 3.

To find the contribution of the saddle point, the path P is

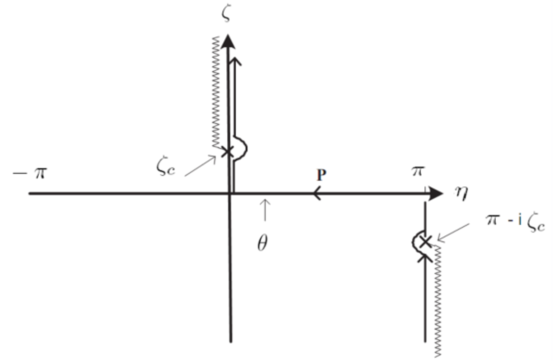

Figure 3. Original Contour of Integration with Branch Cuts in the region $y > 0$.

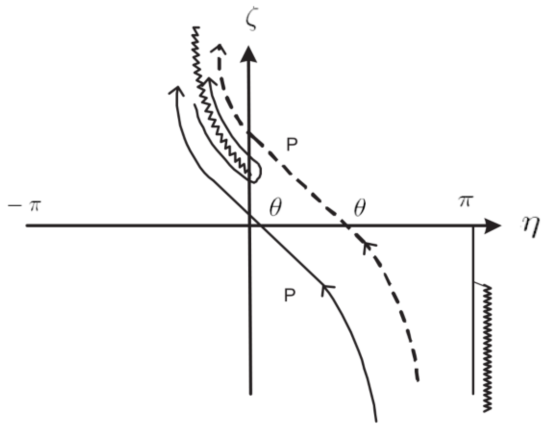

Figure 4. Deformed Contour of Integration (SDP) with Contribution of Branch Cuts in the region $y > 0$.

deformed to the steepest descent path through the saddle point and evaluated with standard methods [8].

The branch point does not contribute in the evaluation of the integral if $\theta > \cos^{-1}(1/n)$. In this case, the steepest descent path avoids the branch cut shown by the dotted line in the Fig. 4. If $\theta > \cos^{-1}(1/n)$, the only contribution to the integral is due to the saddle point. If $\theta < \cos^{-1}(1/n)$, the steepest descent path cannot avoid the branch cut shown as a solid line in Fig. 4. In this case, the contributions due to the saddle point and the branch cut are both taken into account. For this purpose, the branch cut is deformed to coincide with the steepest descent path.

The resulting asymptotic field in the region $0 \le \theta \le \pi/2$ is given as

$$E_{z1} = -\frac{\omega\mu I e^{ik_1 r}}{\sqrt{2\pi k_1 r}}\left\{F_1(\theta)e^{-i\pi/4} + \frac{F_2(\theta)e^{-i3\pi/4}}{2k_1 r}\right\} - \frac{\omega\mu I\sqrt{n}e^{i\left(k_2 r\cos\theta + ik_1 r\sqrt{n^2-1}\sin\theta\right)}e^{-i3\pi/4}}{\sqrt{2\pi}(n^2-1)^{1/4}(k_1 r)^{3/2}(\sqrt{n^2-1}\cos\theta + in\sin\theta)^{3/2}} \times u(n\cos\theta - 1), \quad (8)$$

where

$$\begin{aligned} F_1(\theta) &= \frac{\sin\theta}{(\sin\theta + \sqrt{n^2-\cos^2\theta})}, \\ F_2(\theta) &= \frac{1}{(\sin\theta + \sqrt{n^2-\cos^2\theta})(n^2-\cos^2\theta)^{3/2}} \\ &\times \left[\{2\cos^4\theta + n^2\sin\theta(\sin\theta - \sqrt{n^2-\cos^2\theta}) - 2\cos^2\theta(n^2 - \sin\theta\sqrt{n^2-\cos^2\theta})\} + \frac{\sin\theta(n^2-\cos^2\theta)^{3/2}}{4}\right], \end{aligned} \quad (9)$$

and $u(\cdot)$ is the unit step function. The first term in (8) is due to the saddle point contribution, while the second term is due to the branch point contribution. The saddle point contribution has a leading term which decays as $(k_1 r)^{-1/2}$. The next term decays as $(k_1 r)^{-3/2}$. The branch cut integral contributes a leading term which decays as $(k_1 r)^{-3/2}$. The next term decays as $(k_1 r)^{-5/2}$ and, being smaller than $(k_1 r)^{-3/2}$, has been neglected. It is also seen that the branch cut contribution which is present only when $n\cos\theta > 1$ appears as an exponentially decaying wave above the interface. This decaying wave propagates along the x-axis with the wavenumber of the lower medium.
In a similar manner, (3) is transformed from the k_x plane to the α plane, resulting in

$$E_{z2} = -\frac{\omega\mu I}{2\pi}\int_P \frac{n\sin\alpha}{n\sin\alpha + \sqrt{1-n^2\cos^2\alpha}} \times \exp(ik_2 r\cos(\alpha+\theta))\,d\alpha. \quad (10)$$

The path of integration is shown in Fig. 5. The branch points in this case are located on the real axis in the α plane at $\eta_c = \cos^{-1}(1/n)$ and $\eta_c = \pi - \cos^{-1}(1/n)$. The path is deformed to a steepest descent path which passes through the saddle point located at $\alpha = \theta$, as shown in Fig. 6. It is evident that the branch cut contribution arises only when $\theta < \zeta_c$, shown by the solid line in Fig. 6. The asymptotic evaluation of (10) is carried out similarly to (7), which leads to the following expression for the field in the region $-\pi/2 < \theta < 0$:

$$E_{z2} \simeq -\frac{\omega\mu I e^{ik_2 r}}{\sqrt{2\pi k_2 r}}\left\{F_3(\theta)e^{-i\pi/4} + \frac{F_4(\theta)e^{-i3\pi/4}}{2(k_2 r)}\right\} - \frac{\omega\mu I e^{ik_1 r\left(\cos\theta - \sqrt{n^2-1}\sin\theta\right)}e^{-i3\pi/4}}{\sqrt{2\pi}(n^2-1)^{1/4}(k_1 r)^{3/2}\{\sqrt{n^2-1}\cos\theta + \sin\theta\}^{3/2}} \times u(n\cos\theta - 1), \quad (11)$$

where

$$\begin{aligned} F_3(\theta) &= \frac{\sin\theta}{(\sin\theta + \sqrt{1/n^2-\cos^2\theta})}, \\ F_4(\theta) &= \frac{1}{(\sin\theta + \sqrt{1/n^2-\cos^2\theta})(1/n^2-\cos^2\theta)^{3/2}} \\ &\times \left\{2\cos^4\theta + \frac{1}{n^2}\sin\theta(\sin\theta - \sqrt{1/n^2-\cos^2\theta}) - 2\cos^2\theta(1/n^2 - \sin\theta\sqrt{1/n^2-\cos^2\theta}) + \frac{\sin\theta(1/n^2-\cos^2\theta)^{3/2}}{4}\right\}. \end{aligned} \quad (12)$$

The first term in (11) is due to the saddle point contribution, while the last term is due to the branch point. In the above expression, the terms which decay faster than $(k_{1,2}r)^{-3/2}$ have been omitted. The contribution of the lead-

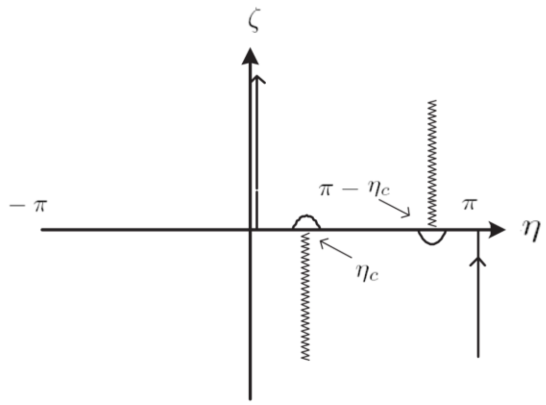

Figure 5. Original Contour of Integration in the region $y < 0$.

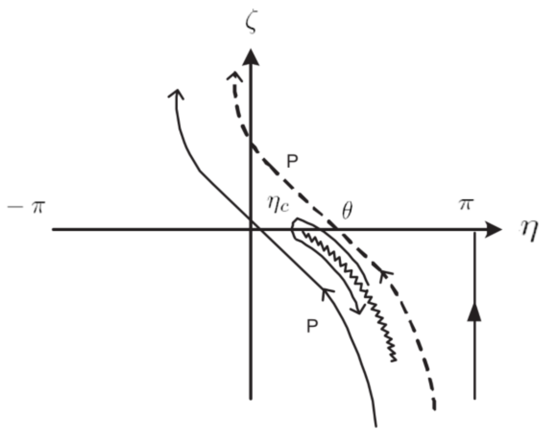

Figure 6. Deformed Contour of Integration (SDP) with $\theta < \eta_c$ in the region $y < 0$.

ing term of the saddle point is again found to be zero along the interface. It can be seen that no evanescent wave decaying perpendicular to the interface exists in the lower medium.

4. Results and conclusions

Asymptotic expressions for the electric field in the regions $y > 0$ and $y < 0$ have been obtained in (8) and (11), respectively. The contributions of different terms present in the overall field expressions will now be discussed.

A very close mathematical symmetry is found to exist between (8) and (11). $F_1(\theta)$ and $F_3(\theta)$ differ only by the reciprocal of refractive index n. The same is also true for $F_2(\theta)$ and $F_4(\theta)$. It is also evident that at the interface $y = 0$, the branch point contribution for E_{z1} equals

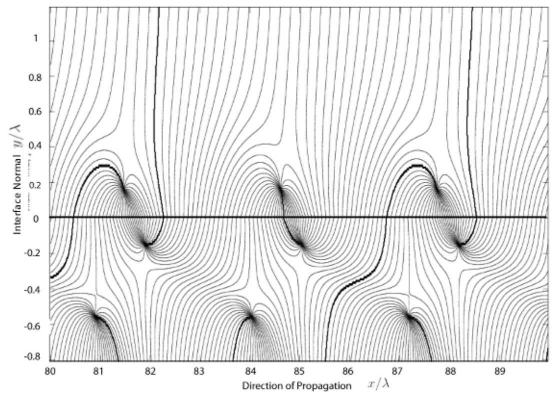

Figure 7. Phase plot of lines of constant phase near the interface.

the saddle point contribution for E_{z2}. The saddle point contribution in E_{z1} at the interface similarly equals the branch point contribution in E_{z2}. The branch point in E_{z1} is responsible for a lateral wave which travels in the faster medium according to the velocity of the slower medium, but which decays exponentially above the interface. This lateral wave in the upper medium is supported by the normal slow wave in the lower medium due to the saddle point contribution in E_{z2}. Similarly, the fast wave in the upper medium supports a similar wave in the lower medium to preserve the continuity of the phase fronts. In the lower half space, the wave due to the branch point contribution in E_{z2} also bends towards the interface normal.

It may be noted that the tangential electric field is continuous across the interface and is non-zero along the interface. It may be verified that the tangential magnetic field up to the term which decays as $(k_{1,2}r)^{-3/2}$ is also non-zero and continuous at the interface. This result is different from [4], which shows a null on the interface and consequently avoids the question of continuity of phase velocity across the interface. The phase velocity of an electromagnetic wave may be defined as

$$\mathrm{v} = \frac{\omega}{|\nabla\phi|^2}\nabla\phi.$$

in analogy with plane waves. The equiphase contours of the electric field will be called phase maps. The lines of phase velocity will be orthogonal to the equiphase contours.

The phase map of the field near the interface is shown in Fig. 7. This phase map shows that there exist some points where phase is being generated. The lines of phase velocity near these points form closed curves and waves seem to circulate around these points. The magnitude of the field at these points is zero. These points are called center type critical points [9]. Above each center point is

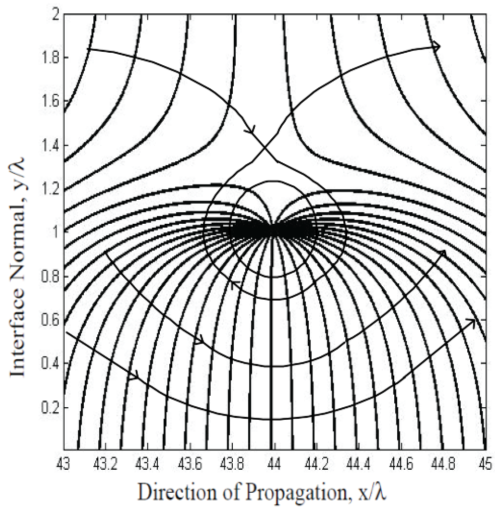

Figure 8. Orthogonal lines drawn on phase map to show saddle and center type of critical points of phase velocity.

another type of point where equiphase contours intersect each other. The lines of phase velocity near these points form hyperbolic curves. These are points of stagnation, and waves seem to avoid flowing through these points. At these points, the electric and magnetic fields are in time quadrature. Hencem, the Poynting vector field is zero at these points. Such points on phase maps are called saddle type critical points [9]. A close-up of the phase map and lines of phase velocity can be seen in Fig. 8. It is observed from this figure that two adjacent arms of the saddle type critical point enclose the center type critical point. The pair of saddle and center type critical points gives rise to an interesting structure. The overall effect of this structure is to slow down the wave as it enters into the second medium, by generating 2π lines of extra phase. It is seen in Fig. 7 that these structures are repeated along the interface boundary.

The location of these interesting structures can be found conveniently by locating the center type critical points. This is done by finding the location of nulls of the electric field, *i.e.*, by solving $E_{z1}(r,\theta) = 0$ for r and θ in theupper half space and $E_{z2}(r,\theta) = 0$ for r and θ in the lower half space. It can be shown that the solution of $E_{z1}(r,\theta) = 0$ and $E_{z2}(r,\theta) = 0$ does not exist when the respective branch point contributions are not included in these field expressions.

Considering the upper half space solution first, let $R = k_1 r$ in (8). The phase term of the branch point contribution part in (8), can be written as $R(n\cos\theta - 1) - \psi \approx R(n\cos\theta - 1)$, where

$$\psi(\theta) = \frac{3}{2}\tan^{-1}\left(\frac{n\tan\theta}{\sqrt{n^2-1}}\right),$$

when $R \gg 1$. It can be shown that in the absence of the branch point contribution in the evaluated field, the solution for the nulls does not exist in (8). So it can be said that when $\zeta_c < \theta < \pi/2$, there is no solution for the null. On the other hand, the solution for the null exists when both saddle point and branch point contributions are included in the evaluated field expression,or in the case when $0 < \theta < \zeta_c$.

The solution for $E_{z1}(r,\theta) = 0$ in the upper half space is obtained by taking the first term of the Taylor series expansion near $\theta = 0$ in (8). By equating the real and imaginary parts of these terms, respectively, the following simultaneous equations in $R = k_1 r$ and θ are obtained:

$$\cos\{R(n-1)\} = \frac{1}{\sqrt{n}} e^{R\theta\sqrt{n^2-1}} = \frac{e^u}{\sqrt{n}} \tag{13}$$

$$\sin\{R(n-1)\} = -\frac{R\theta\sqrt{n^2-1}}{\sqrt{n}} e^{R\theta\sqrt{n^2-1}} = -\frac{u}{\sqrt{n}} e^u, \tag{14}$$

where

$$u = R\theta\sqrt{n^2-1}.$$

The above equations are squared and added to obtain the following transcendental equation:

$$1 + u^2 = n e^{-2u}.$$

This transcendental equation can be solved for u using fixed point iterations of the following form:

$$u_{j+1} = -\frac{1}{2}\ln\left(\frac{1+u_j^2}{n}\right),$$

where j is the index. For a given value of n, the right hand sides of Eqs. (13) and (14) are fixed. Thus, there are many values of R and θ which satisfy these simultaneous equations. Hence the locations of center points are given as:

$$R_m = \frac{2m\pi - \sin^{-1}\left(\frac{u}{\sqrt{1+u^2}}\right)}{n-1}, \qquad \theta_m = \frac{u}{R_m\sqrt{n^2-1}}, \tag{15}$$

where m is a positive integer.

The points where the electric field has nulls in the lower half space are determined following the same procedure as for E_{z1} in the upper half space.

Equating the real and imaginary parts of the leading term of the Taylor series expansion, the following solution is obtained for $E_{z2}(r,\theta)=0$:

$$R_m = \frac{m\pi}{n-1},$$
$$\theta_m = \frac{\sqrt{1-\frac{1}{n^2}}}{R}\left\{\frac{\sqrt{n}}{n^2-1}(\pm 1)^m - \frac{1}{n(1-\frac{1}{n^2})}\right\}. \quad (16)$$

$$R\delta\theta = \frac{(\pm 1)^m}{\sqrt{n}\sqrt{n^2-1}} - \frac{1}{\sqrt{n^2-1}} = \frac{1}{\sqrt{n^2-1}}\left\{\frac{(\pm 1)^m}{\sqrt{n}} - 1\right\}.$$

From the above expression, it is clear that for a given constant height from the interface,critical points exist in the form of a pair. Their location is also dependent on the refractive index n of the medium.

Thus it can be concluded that the center type critical points repeat along the interface. The number of critical points per unit length, normalized with respect to wavelength in the rarer mediumL/λ_1 can be easily shown to be $(n-1)$. If only the leading term in the asymptotic expression for E_{z1}, *i.e.*, the saddle point term is retained, then the field at the interface vanishes. This is a structurally unstable condition [7]. This instability is removed by perturbation coming from leading terms of branch cuts which decay as $(k_{1,2}r)^{-1}$ in (8). The resulting structure of center type critical points along with saddle type critical points which is repeated along the interface is a structurally stable situation. It has been observed that no qualitative change in phase map occurs in the far zone along the interface if terms of order higher than $(k_{1,2}r)^{-3/2}$ are included in (8).

Finally, it can be concluded that the phase velocity of the wave in the rarer medium decreases in a continuous manner to match the phase velocity of the wave in the lower medium. This is made possible due to the presence of saddle and center type of critical points in the phase map. The phase map is not only continuous but also structurally stable.

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