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Einstein–Hopf drag, Doppler shift of thermal radiation and blackbody drag: Three perspectives on quantum friction

Research Article

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Abstract: The thermal friction force acting on an atom moving relative to a thermal photon bath has recently been calculated on the basis of the fluctuation-dissipation theorem. The thermal *fluctuations* of the electromagnetic field give rise to a drag force on an atom provided one allows for *dissipation* of the field energy via spontaneous emission. The drag force exists if the atomic polarizability has a nonvanishing imaginary part. Here, we explore alternative derivations. The damping of the motion of a simple harmonic oscillator is described by radiative reaction theory (result of Einstein and Hopf), taking into account the known stochastic fluctuations of the electromagnetic field. Describing the excitations of the atom as an ensemble of damped harmonic oscillators, we identify the previously found expressions as generalizations of the Einstein-Hopf result. In addition, we present a simple explanation for blackbody friction in terms of a Doppler shift of the thermal radiation in the inertial frame of the moving atom: The atom absorbs blue-shifted photons from the front and radiates off energy in all directions, thereby losing energy. The original plus the two alternative derivations provide for additional confirmation of an intriguing quantum friction effect, and leave no doubt regarding its existence.

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1. Introduction

Initially, one might conjecture that the influence of incoming electromagnetic radiation on an atom or ion should be restricted to the well-known conservative radiation pressure force, and not involve drag or friction. When averag-

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ing the radiation pressure force of incoming electromagnetic radiation on an atom over all directions, the effect should vanish for spatially uniform blackbody radiation. However, on second thought, the existence of such a drag force is plausible. The reason is that the fluctuations of the electromagnetic field naturally lead to fluctuations of the atomic dipole moment, which in turn lead to the dissipation of energy provided the atom undergoes spontaneous emission of radiation. According to Ref. [1], the friction coefficient can be calculated based on a Green–Kubo formula, which amounts to an integration over all possible points in time at which the dissipative process may occur.

As a consequence, a particle moving through a thermal radiation field experiences this dissipation as a drag force. The drag force acting on an atom flying through space with a velocity v with respect to the cosmic microwave background (CMB) rest frame is nonvanishing. Already in 1910, soon after the discovery of Planck's formula for the thermal distribution of light quanta emerging from a perfect black body, this friction force was calculated by Einstein and Hopf [2], for the simplified case of a single harmonic oscillator subject to radiative damping. In the simple model considered by Einstein and Hopf, the Abraham–Lorentz radiation reaction force determines the "width" term for the damped harmonic motion. Because an atom can be viewed as an ensemble of damped harmonic oscillators, it is natural to look for possible connections of the historic result presented by Einstein and Hopf with the recent derivation presented in Ref. [1].

A totally different point of view is reached if we interpret the direction-dependent drag force in terms of a direction-dependent Doppler effect: The atom sees blue-shifted, incoming photons from the front, whereas incoming photons from the back are red-shifted, while the atom radiates off the photons in all possible directions. This leads to a drain on the energy of the atom, which corresponds to a friction force. The Doppler shifted, direction-dependent temperature of blackbody radiation has been calculated in Ref. [3]. Incident radiation exerts a force on an atom, proportional to the imaginary part of the atomic polarizability [4]. The precise connection of these observations with the result derived in Ref. [1] is clarified in the following. The two alternative derivations of the blackbody friction force sketched above confirm the existence of the quantum blackbody friction effect.

SI mksA units are used throughout the article. We proceed as follows. The connection to the Einstein–Hopf drag is explored in Sec. 2, while an alternative derivation based on the Doppler effect of the thermal radiation is given in Sec. 3. Conclusions are drawn in Sec. 4.

2. Einstein–Hopf drag

Roughly a century ago, Einstein and Hopf [2] considered a classical charged particle, having charge e and mass m, moving in a one dimensional harmonic potential, and being in thermal equilibrium with the electromagnetic field. The equation of motion for this particle, including the Abraham-Lorentz radiative reaction force and the coupling to the electric field $\mathcal{E}_x$, is given by:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega_0^2 \, x - \sigma \frac{\mathrm{d}^3 x}{\mathrm{d}t^3} = \frac{e}{m} \mathcal{E}_x(t) \,, \tag{1}$$

with $\sigma = e^2/(6\pi\epsilon_0 \, m c^3)$. The condition of thermal equilibrium between the atom and the thermal bath implies a nonvanishing Einstein–Hopf (EH) damping force $F_{\rm EH}$ if the center of mass of the harmonic potential is in motion with respect to the field. In the limit of slow motion, the velocity-dependent force is equal to

$$F_{\rm EH} = -\frac{4\pi^2 e^2 \, v}{5 m c^2 \, (4\pi\epsilon_0)} \left(\rho(\omega_0; T) - \frac{\omega_0}{3} \frac{\partial \rho(\omega_0; T)}{\partial \omega_0} \right) \,, \tag{2}$$

a result which has been recorded in Ref. [2] and in Eq. (4) in Ref. [5]. Herein, $\rho(\omega; T)$ represents the energy density of the electromagnetic radiation,

$$\rho(\omega; T) = \frac{\hbar \, \omega^3}{\pi^2 \, c^3} \, \eta(\omega; T) \,. \tag{3}$$

We now use the identity

$$\rho(\omega; T) - \frac{\omega}{3} \frac{\partial \rho(\omega; T)}{\partial \omega} = -\frac{\hbar \, \omega^4}{3\pi^2 \, c^3} \frac{\partial \eta(\omega; T)}{\partial \omega} \,, \tag{4}$$

$$\eta(\omega; T) = \frac{1}{\exp\left(\beta \hbar \omega\right) - 1} \,, \tag{5}$$

where $\eta(\omega; T)$ is the thermal photon occupation number. So,

$$F_{\rm EH} = \frac{4 \hbar \, e^2 \, \omega_0^4 \, v}{15 \, m c^2 \, (4\pi\epsilon_0)} \frac{\partial \eta(\omega_0; T)}{\partial \omega_0} = \frac{\hbar \, e^2 \, \omega_0^4 \, v}{15 \, \pi \, m c^2 \, \epsilon_0} \frac{\partial \eta(\omega_0; T)}{\partial \omega_0} \,. \tag{6}$$

The connection to Ref. [1] can now be found as follows. According to Refs. [6, 7], the polarizability tensor is isotropic for atoms, and the main result in Ref. [1] is an expression for the effective friction (EF) force as the spectral integral of the imaginary part of the dynamic dipole polarizability of the particle [see Eq. (12) in Ref. [1]]:

$$\begin{aligned} F_{\rm EF} &= -\frac{\beta \hbar^2 \, v}{3\pi \, c^5 \, (4\pi\epsilon_0)} \int_0^\infty \mathrm{d}\omega \, \frac{\omega^5 \, {\rm Im} \, \alpha(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)} \\ &= \frac{\hbar \, v}{3\pi^2 \, c^5 \, \epsilon_0} \int_0^\infty \mathrm{d}\omega \, \omega^5 \, \frac{\partial \eta(\omega; T)}{\partial \omega} \, {\rm Im} \, \alpha(\omega) \,, \end{aligned} \tag{7}$$

Here, $\beta = 1/(k_B T)$ is the Boltzmann factor and $\alpha(\omega)$ represents the dynamic polarizability. Note that $F_{\rm EF}$ points in a direction opposite to the velocity v of the particle and thus acts as a drag force; the sign of the derivative $\partial\eta(\omega;T)/\partial\omega$ is negative. We have used the identity

$$\frac{\beta\hbar}{\sinh^2(\frac{1}{2}\beta\hbar\omega)} = -4\,\frac{\partial\eta(\omega;T)}{\partial\omega}\,. \tag{8}$$

In order to compare the results in Eqs. (6) and Eq. (7), we follow an argument presented in Ref. [5]: For randomly oriented, classical oscillators, the formula (7) needs to be averaged over the projection of the electromagnetic wave vector $\vec{k}$ onto the z-axis (along which the potential center is moving), and over the projections of the polarization vectors $\vec{\epsilon}_{\vec{k}\lambda}$ onto the x-axis (the direction of harmonic oscillation), and over the photon polarizations. This gives an additional factor of

$$\frac{1}{2}\int\frac{d\Omega_k}{4\pi}\,\hat{k}_z^2\sum_\lambda \epsilon^2_{\vec{k}\lambda x} = \frac{2}{15}\,. \tag{9}$$

The forces calculated using Eqs. (6) and (7) can be compared by solving the classical equation of motion for the Einstein-Hopf model with an oscillatory electric field. The corresponding classical dynamic polarizability reads as follows (HO stands for a harmonic oscillator)

$$\alpha_{\rm HO}(\omega) = \frac{e^2}{m}\,\frac{1}{\omega^2-\omega_0^2-\mathrm{i}\,\sigma\,\omega^3}\,, \tag{10}$$

which is proportional to the Green function for the oscillator described by Eq. (1). In the limit of small radiative damping, i.e. $\sigma \ll 1/\omega_0$, the nonvanishing values of $\alpha_{\rm HO}(\omega)$ is concentrated in the area $\omega \approx \omega_0$, and we can approximate

$$\mathrm{Im}\,\alpha_{\rm HO}(\omega) \approx \frac{\pi e^2}{2\omega_0\,m}\,\delta(\omega-\omega_0)\,. \tag{11}$$

The matching of the atomic polarizability against that of a harmonic oscillator now proceeds as follows,

$$\mathrm{Im}\,\alpha(\omega) = \frac{2}{15}\cdot 3\cdot \mathrm{Im}\,\alpha_{\rm HO}(\omega) = \frac{\pi\,e^2}{5\,m\,\omega_0}\,\delta(\omega-\omega_0)\,. \tag{12}$$

The factor 2/15 stems from Eq. (9). The factor 3 is due to the fact that the atomic polarizability (for the ground state) is isotropic, and therefore the factor 3 counts the possible excitations in all three directions of space. The identification of $F_{\rm EF}$ as $F_{\rm EH}$ is now done easily,

$$\begin{aligned} F_{\rm EF} =& \frac{\hbar\,v}{3\pi^2\,c^5\,\epsilon_0}\int_0^\infty d\omega\,\omega^5\,\frac{\partial\eta(\omega;T)}{\partial\omega}\,\mathrm{Im}\,\alpha(\omega) \to \\ & \frac{\hbar\,v}{3\pi^2\,c^5\,\epsilon_0}\int_0^\infty d\omega\,\omega^5\,\frac{\partial\eta(\omega;T)}{\partial\omega}\,\frac{\pi\,e^2}{5\,m\,\omega_0}\,\delta(\omega-\omega_0) \to \\ & \frac{\hbar\,e^2\,\omega_0^4\,v}{15\,\pi\,m\,c^5\,\epsilon_0}\,\frac{\partial\eta(\omega_0;T)}{\partial\omega_0} = F_{\rm EH}\,. \end{aligned} \tag{13}$$

This shows that the Einstein–Hopf and the effective-damping force have the same physical origin and describe the same physical effect.

3. Doppler effect and blackbody friction

In order to derive the blackbody friction force on an atom based on the direction-dependent Doppler effect, we need (i) a result which allows us to express the shift in the incoming blackbody radiation frequencies in terms of a direction-dependent temperature, and (ii) a result which connects the incoming light intensity to the imaginary (dissipative) part of the atomic polarizability.

The second of these prerequisites is given in Ref. [3]. Indeed, the combined effects of the Doppler shift and the change in intensity due to Lorentz contraction [3], visible for a moving observer, leads to a change of the observed temperature T_o of the thermal bath, which depends on the relative angle θ between the wave vector and the velocity $\vec{v}$ and the magnitude of the latter:

$$T_o(\theta) = T\sqrt{\frac{c-v\cos\theta}{c+v\cos\theta}} = T - T\,\frac{v}{c}\cos\theta + \mathcal{O}(v^2)\,. \tag{14}$$

This result is based on the known properties of blackbody radiation under Lorentz transformation [3].

The first prerequisite can be found in Ref. [4]. Namely, the force acting on a particle in the field of plane monochromatic radiation of intensity $I(\omega)$ is equal to

$$F = -\frac{\omega}{\epsilon_0\,c}\,\mathrm{Im}\,\alpha(\omega)\,I(\omega)\,. \tag{15}$$

In the case of a particle moving through blackbody radiation, the direction-dependent intensity distribution is

$$I(\omega) = \rho(\omega;T_o(\theta))/(4\pi)\,. \tag{16}$$

The rest of the calculation is rather straightforward. A particle moving through blackbody radiation experiences a nonzero force parallel to its velocity. In view of the above, we have

$$F \sim \int_0^\infty d\omega \left(-\frac{\omega}{\epsilon_0\, c} \operatorname{Im} \alpha(\omega) \right) \int d\Omega \, \cos\theta \, \frac{\rho\big(\omega; T_\circ(\theta)\big)}{4\pi} \,. \tag{17}$$

Using the chain rule, we have

$$\begin{aligned} \rho\big(\omega; T_\circ(\theta)\big) =& \rho\big(\omega; T - T\,\frac{v}{c}\cos\theta\big) \approx \\ & \rho\big(\omega; T\big) + \frac{\partial}{\partial T}\rho\,(\omega; T)\, \left(-T\,\frac{v}{c}\cos\theta \right) \,. \end{aligned} \tag{18}$$

Concerning the sign of the force, we note that for positive driving frequencies, the imaginary part of the atomic polarizability in Eq. (15) is positive. The force F given in Eq. (15) is thus negative and slows the atom. The derivative $\partial\rho\,(\omega; T)\,/\partial T$ is positive. We know that the effective temperature of blackbody radiation has to increase in the flight direction of the atom, i.e., when $\theta = 0$. By contrast, in formula (14), θ measures the angle with respect to the incoming radiation, not the angle with respect to the counterpropagating direction. We thus insert a minus sign and extract the velocity-dependent part of Eq. (14) as follows,

$$\begin{aligned} F =& \int_0^\infty d\omega \left(-\frac{\omega}{\epsilon_0\, c} \operatorname{Im} \alpha(\omega) \right) \\ & \int \frac{d\Omega}{4\pi} \cos\theta \left(\frac{\partial}{\partial T}\rho\,(\omega; T) \right) \left(+T\,\frac{v}{c}\cos\theta \right) \\ =& -\frac{v}{3\epsilon_0\, c^2} \int_0^\infty d\omega\, \omega\, T\, \frac{\partial \rho(\omega; T)}{\partial T} \operatorname{Im} \alpha(\omega) \\ =& -\frac{v}{3\epsilon_0\, c^2} \int_0^\infty d\omega\, \omega \left(-\frac{\hbar\,\omega^4}{\pi^2\, c^3}\, \frac{\partial \eta(\omega; T)}{\partial\omega} \right) \operatorname{Im} \alpha(\omega) \\ =& \frac{\hbar v}{3\,\pi^2\, c^5\, \epsilon_0} \int_0^\infty d\omega\, \omega^5\, \frac{\partial \eta(\omega; T)}{\partial \omega} \operatorname{Im} \alpha(\omega)\,, \end{aligned} \tag{19}$$

which is exactly equal to Eq. (7). We have used the identity

$$T\,\frac{\partial \rho(\omega; T)}{\partial T} = -\frac{\hbar\,\omega^4}{\pi^2\, c^3}\, \frac{\partial \eta(\omega; T)}{\partial \omega}\,, \tag{20}$$

which is elementary. We are thus able to invoke a simple picture, in which the drag introduced in (7) can ultimately be traced back to the Doppler effect. A priori, the stochastic field might be thought to increase the kinetic energy of the classical particle, because the fluctuations continuously kick the particle. Once the particle is in motion, however, the thermal equilibrium condition leads to a decrease in the kinetic energy down to the recoil limit. Thermal photons approaching the moving particle from the front are blue-shifted, whereas photons coming in from the back are red-shifted and thus less energetic. In contrast, the emission is symmetric with respect to the forward and backward directions in the rest frame of atom. At thermal equilibrium between the moving particle and the fluctuating field we thus have a net dissipation of energy, experienced as drag, or blackbody friction.

4. Conclusions

We provide two alternative derivations for the blackbody friction force on a moving atom, originally derived in Ref. [1]. In particular, we show that the Einstein–Hopf result for thermal friction can be interpreted in terms of the thermal friction force on an atom, provided the atomic polarizability tensor is replaced by the one corresponding to a classical object under the influence of a damping force, that is proportional to the time derivative of the acceleration (radiative reaction).

We also present an intuitive picture, in which the blackbody friction force is ascribed to a change in temperature of the thermal bath, as observed by an atom moving through it, for the thermal photons coming from different directions. Indeed, the *combined* effect of the Lorentz contraction and the corresponding change in intensity of the blackbody radiation and of the Doppler shift can be absorbed in a direction-dependent temperature change for the thermal photons. Expanding the temperature change to first order in the velocity, we give an immediate and straightforward rederivation of the effect.

We note that the general physical basis for quantum friction effects has recently been intensively discussed in the literature (see Refs. [8–12]). As clarified in Ref. [13], all three derivations discussed here are valid for a dilute system, e.g., an isolated atom and would have to be modified for condensed matter where the additional field due to the polarization of the medium plays a role. However, these modifications are irrelevant in the astrophysical context [1].

We hope that these considerations not only lead to a confirmation of the expressions used earlier for the evaluation of black-body friction [1], but also to a deeper understanding of this phenomenon. Last, but not least, let us mention that the precise numerical evaluation of the integral expression given in Eq. (6) leads to very interesting problems, both on the conceptual as well as on the numerical side. These are discussed in Ref. [14]. Indeed, it turns out that the numerical evaluation of the integral

$$F_{\rm EF} = -\frac{\beta \hbar^2 v}{3\pi c^5 (4\pi\epsilon_0)} \int_0^\infty \mathrm{d}\omega \, \frac{\omega^5 \,\mathrm{Im}\, \alpha(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)} \,, \qquad (21)$$

is highly nontrivial for medium and small temperatures: The thermal factor (the hyperbolic sine) in the denominator drops so fast with the frequency that the tail of the Lorentzian line curves in the polarizability become important. This leads to a very peculiar behaviour of the friction force as a function of the temperature, as outlined in Ref. [14].

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