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Center-symmetric effective theory for two-color quark matter

Short Communication

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Abstract: We revisit the center-symmetric dimensionally reduced effective theory for two-color Yang–Mills theory at high temperature. This effective theory includes an order parameter for deconfinement and thus allows to broaden the range of validity of the conventional three-dimensional effective theory (EQCD) towards the confining phase transition. We extend the previous results by including the effects of massive quarks with nonzero baryon chemical potential. The parameter space of the theory is constrained by leading-order matching to the Polyakov loop effective potential of two-color QCD. Once all the parameters are fixed, the effective theory can provide model-independent predictions for the physics above the deconfinement transition, thus bridging the gap between large-scale numerical simulations and semi-analytical calculations within phenomenological models.

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1. Introduction

Dimensionally reduced effective field theories provide an efficient approach to thermodynamics of strongly interacting matter at high temperatures. They rely essentially on the separation of physical scales: the hard scale of the order of the temperature T, and the soft scale of order gT where g is the dimensionless coupling of the theory. Nevertheless, the applicability of the dimensionally reduced effective theory of QCD, the electrostatic QCD (EQCD), to phenomenologically interesting temperatures is seriously limited by the fact that it does not respect the center symmetry which is crucial for the confinement phase transition.

In Ref. [1], an extension of EQCD was proposed for the three-color Yang–Mills theory which incorporates the center symmetry as a basic building block. The resulting center-symmetric dimensionally reduced effective theory was dubbed ZQCD. This approach was subsequently applied to two-color Yang–Mills theory which is technically considerably easier to deal with [2].

The goal of the present contribution is to extend the results of Ref. [2] by including the effects of dynamical quarks of in principle arbitrary masses and chemical potentials. This opens the possibility to use ZQCD to study the thermodynamics of the phase diagram of two-color QCD. At the same time, we correct an algebraic error in the previ-

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ous paper [2] which resulted in a wrong matching condition for the ZQCD couplings.

2. The effective theory

The order parameter in (two-color) QCD related to the deconfinement phase transition, and thus spontaneous breaking of center symmetry, is the expectation value of the Polyakov loop. In ZQCD, this is implemented by means of an effective field $\mathcal{Z}(\mathbf{x})$ which can be interpreted as the Polyakov loop coarse-grained over length scales of order $1/T$. The prime technical advantage of two-color QCD as opposed to the three-color case is that this coarse-graining procedure turns a unitary matrix into one which is unitary up to a multiplicative real factor. We therefore parameterize $\mathcal{Z}(\mathbf{x})$ as $\mathcal{Z} = (\Sigma + \mathrm{i}\vec{\Pi}\cdot\vec{\sigma})/2$ where σ_a are the Pauli matrices. Taking in addition into account the three-dimensional gluon field $\vec{A}(\mathbf{x})$, we construct the most general Lagrangian density with operators up to fourth order in the fields, compatible with the underlying SU(2) gauge invariance,

$$\mathcal{L} = \frac{1}{g_3^2}\Big[\frac{1}{2}\mathrm{tr}F_{ij}^2 + \mathrm{tr}\big(D_i\mathcal{Z}^\dagger D_i\mathcal{Z}\big) + b_1\Sigma^2 + b_2\vec{\Pi}^2 + c_1\Sigma^4 + c_2(\vec{\Pi}^2)^2 + c_3\Sigma^2\vec{\Pi}^2 + d_1\Sigma^3 + d_2\Sigma\vec{\Pi}^2\Big], \tag{1}$$

where g_3 is the three-dimensional coupling, D_i the (adjoint) covariant derivative, and $F_{ij} \equiv \partial_i A_j - \partial_j A_i - \mathrm{i}[A_i, A_j]$. Note that unlike all the other terms in the Lagrangian, the operators proportional to the couplings $d_{1,2}$ break the Z_2 center symmetry of two-color Yang–Mills theory, defined by $\mathcal{Z} \to \pm\mathcal{Z}$. These operators are induced on the ZQCD level by the presence of dynamical quarks. Later, three of the four degrees of freedom of ZQCD will be identified with the adjoint Higgs field $\vec{A}_0$ of EQCD with its mass of order gT, while the remaining heavy degree of freedom represents an auxiliary mode necessary to ensure (super)renormalizability of the effective Lagrangian (1). The splitting of the mass scales of these modes is accomplished by rewriting the effective couplings in terms of 'hard' and 'soft' parts, $b_1 = \frac{1}{2}h_1$, $b_2 = \frac{1}{2}(h_1 + g_3^2 s_1)$, $c_1 = \frac{1}{4}h_2 + g_3^2 s_3$, $c_2 = \frac{1}{4}(h_2 + g_3^2 s_2)$, $c_3 = \frac{1}{2}h_2$, $d_1 = \frac{1}{2}g_3^2 s_4$, $d_2 = \frac{1}{2}g_3^2 s_5$. The hard part of the Lagrangian is then invariant under an extended SU(2) × SU(2) symmetry, which is spontaneously broken by the nonzero vacuum expectation value of $\mathcal{Z}$ to an SU(2) subgroup. This naturally leads to three light modes in the spectrum which only receive their masses from the soft parts of the couplings.

2.1. Perturbative matching to EQCD

In view of the fact that the center symmetry is spontaneously broken at high temperatures, we reparameterize the field $\mathcal{Z}$ as $\mathcal{Z} = \frac{v}{2} + \frac{g_3}{2}(\phi + \mathrm{i}\vec{\chi}\cdot\vec{\sigma})$ where v is the anticipated vacuum expectation value. The adjoint scalar field $\vec{\chi}$ is naturally identified with the adjoint scalar $\vec{A}_0$ of EQCD. Comparing their center transformation properties leads to the relation $v = 2T$ up to corrections of order g^2. Solving to the same order for the minimum of the static part of the Lagrangian (1) then yields the first matching condition,

$$h_1 + 4T^2 h_2 = 0. \tag{2}$$

In order to determine the values of the soft parameters s_i we match the Lagrangian of our effective theory perturbatively to EQCD. This is achieved by integrating out the heavy auxiliary mode ϕ and Taylor expanding in powers of $\vec{\chi}$ to obtain the couplings of the mass and quartic terms for $\vec{\chi}$. The EQCD values of the same couplings can be obtained by Taylor expanding the one-loop center-symmetric effective potential of QCD derived by Weiss [3]. A comparison gives the following two matching conditions,

$$\begin{aligned} s_1 - 4s_3v_0^2 - \frac{3}{2}s_4v_0 + s_5v_0 &= \frac{2T}{3} - \frac{T\kappa_0^-}{\pi^2}, \\ 2s_2 + 8s_3 + \frac{3s_4}{2v_0} - \frac{2s_5}{v_0} &= \frac{2}{3\pi^2 T} + \frac{\kappa_2^-}{12\pi^2 T}, \end{aligned} \tag{3}$$

where the constants $\kappa_\ell^\pm$, depending on the quark masses and chemical potentials m_j and μ_j, are defined by

$$\kappa_\ell^\pm = \sum_{j=1}^{N_f}(\beta m_j)^2\sum_{n=1}^{\infty}(\pm1)^n n^\ell K_2(n\beta m_j)\cosh(n\beta\mu_j). \tag{4}$$

One may worry about the fact that we obtained just two matching conditions for the three parameters $s_{1,2,3}$. However, this is in accord with the general spirit of effective field theory: once the heavy mode ϕ is integrated out, the ZQCD fields satisfy the "unitarity" constraint $\Sigma^2 + \vec{\Pi}^2 = 1$. Thus, out of the operators corresponding to $s_{1,2,3}$, only two are linearly independent. The remaining linear combination of $s_{1,2,3}$ that is left unconstrained by EQCD has no effect on low-energy physics at the soft scale gT.

2.2. Center-symmetry breaking parameters

As EQCD violates the center symmetry explicitly, the values of the parameters $s_{4,5}$ clearly cannot be found by matching to it. Instead, one has to consider the global structure of the Weiss potential [3]. Owing to the Nielsen theorem, gauge-fixing independent matching conditions can be obtained by concentrating on its stationary points. A natural measure of explicit Z_2 symmetry breaking is then the difference of energy density of the global and the metastable minimum of the potential. This is, however, only sensitive to s_4. In order to fix the last parameter, s_5, one can use for instance the difference of the mass squared parameters (curvatures) at the two minima of the potential. All in all, we deduce two matching conditions for the parameters $s_{4,5}$,

$$s_4 = \frac{1}{2\pi^2}(\kappa^-_{-2} - \kappa^+_{-2}), \qquad 2s_5 - 3s_4 = \frac{1}{2\pi^2}(\kappa^+_0 - \kappa^-_0). \quad (5)$$

This concludes the matching of all the parameters of the effective theory necessary to describe the physics at the soft scale gT. The remaining, as yet undetermined parameters can be fixed by a nonperturbative simulation. Nevertheless, they appear to have little effect on the low-energy physics [2].

3. Applications and outlook

In order to test the center symmetry properties of the effective theory, we consider extended field configurations which probe the global structure of the potential for the Polyakov loop. In the limit of exact Z_2 symmetry, the theory exhibits a one-dimensional domain wall solution that interpolates between the two Z_2 minima of the effective potential. Numerical solution of the classical equation of motion for the gauge field yields a domain wall [4] with a tension roughly 9% smaller than the corresponding value in QCD, found analytically in Ref. [5]. In presence of dynamical quarks, the domain wall is no longer stable, but there is still a static three-dimensional spherically symmetric configuration, corresponding to a bubble of the stable vacuum in a metastable environment. In the limit of weak explicit breaking of the center symmetry (large quark masses), the bubble solution can be found in the thin-wall approximation: its profile is given by the one-dimensional domain wall, and the radius is only sensitive to the energy density difference of the stable and the metastable vacuum.

In summary, we have extended the center-symmetric dimensionally reduced effective theory for two-color QCD [2] by including the effects of dynamical quarks. As a byproduct, we corrected an algebraic error in the previous work that appeared in the matching condition for the quartic coupling of EQCD, analogous to our Eq. (3). Now that the relevant effective theory parameters are completely fixed, the theory can be used to make predictions. Although two-color QCD has no sign problem, and thus can be simulated on the lattice without essential difficulties, we still believe ZQCD can provide a useful semi-analytic insight into the thermodynamics around and above the deconfinement transition. One particularly interesting direction to follow would be for instance the analytic continuation of the phase diagram to imaginary chemical potentials. Unlike EQCD, our effective theory should be able to describe the Roberge–Weiss phase transition, owing to its center symmetry. A more thorough discussion of the applications as well as some details of the computations can be found elsewhere [4].

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References

[1] A. Vuorinen, L. G. Yaffe, Phys. Rev. D 74, 025011 (2006)

[2] P. de Forcrand, A. Kurkela, A. Vuorinen, Phys. Rev. D 77, 125014 (2008)

[3] N. Weiss, Phys. Rev. D 24, 475 (1981)

[4] T. Zhang, T. Brauner, A. Kurkela, A. Vuorinen, J. High Energy Phys. 1202, 139 (2012)

[5] T. Bhattacharya, A. Gocksch, C. Korthals Altes, R. D. Pisarski, Phys. Rev. Lett. 66, 998 (1991)