Contents

Preface — v

Habib Ammari
Differential electromagnetic imaging — 1
1 Introduction — 1
2 Basic theory of electromagnetic waves — 3
2.1 The Helmholtz equation — 3
2.2 The Maxwell equations — 3
2.3 Fundamental solutions and radiation conditions — 4
2.4 Transmission and boundary conditions — 5
2.5 Dirichlet and Neumann functions and the Hodge decomposition — 6
2.6 Trace theorems and first Green identity — 7
2.7 Lippman–Schwinger representation formulas — 8
2.8 The Helmholtz–Kirchhoff theorems — 9
2.9 Limiting models — 10
2.10 The Maxwell equations with axis invariance — 11
2.11 The Maxwell equations versus the Helmholtz equation — 12
3 Electric and magnetic polarization tensors — 12
4 small-volume expansions — 13
4.1 The full Maxwell equations — 13
4.2 The eddy currents model — 17
4.3 The Helmholtz equation — 18
4.4 The conductivity equation — 18
4.5 Asymptotic formulas in the time domain — 18
5 Imaging in the frequency domain — 19
5.1 MUSIC-type imaging at a single frequency — 20
5.2 Backpropagation type imaging at a single frequency — 22
5.3 Imaging with a broad range of frequencies — 23
6 Imaging in the time domain — 24
6.1 Time-domain imaging with full view measurements — 24
6.2 Time-domain imaging in a cavity with limited-view data — 25
6.3 Time-domain imaging in dissipative media — 28
7 Numerical examples of MUSIC reconstructions for the full Maxwell equations — 33
8 Shape representations — 38
8.1 High-order polarization tensors — 38
8.2 Frequency dependent high-order polarization tensors — 41
9 Far-field imaging versus near-field imaging — 45
10 Open problems — 47
Xavier Claeys, Ralf Hiptmair and Carlos Jerez-Hanckes

**Multitrace boundary integral equations — 51**

1 Introduction — 51
1.1 Geometry — 54
1.2 Transmission problems — 54
2 Boundary integral operators — 57
2.1 Trace spaces and operators — 58
2.2 Potentials — 61
2.3 Calderón projectors — 62
3 Classical single-trace integral equations — 64
3.1 Skeleton trace spaces — 65
3.2 A first-kind boundary integral equation — 69
3.3 Boundary element Galerkin discretization — 72
4 Preconditioning — 75
4.1 Operator products — 76
4.2 Calderón identities — 77
4.3 Operator preconditioning — 79
4.4 Stable duality pairing for boundary elements — 80
4.5 The challenge — 81
5 Global multitrace formulation — 82
5.1 Separated subdomains — 82
5.2 The gap idea — 85
5.3 Properties of global MTF — 86
5.4 Galerkin discretization — 87
6 Local multitrace formulation — 88
6.1 Partial transmission conditions — 89
6.2 Local MTF: variational formulation — 91
6.3 Local MTF: Stability — 93
6.4 Boundary element Galerkin discretization — 95

Johannes Elschner and Guanghui Hu

**Direct and Inverse Elastic Scattering Problems for Diffraction Gratings — 101**

1 Introduction — 101
2 Mathematical formulation of direct and inverse scattering problems — 103
3 Solvability results for direct scattering problems: variational method — 107
3.1 An equivalent variational formulation and its Fredholm property — 107
3.2 Uniqueness and existence for direct scattering problems — 109
3.3 Uniqueness and existence for transmission gratings — 111
4 Uniqueness for inverse scattering problems — 115
### Oliver G. Ernst and Martin J. Gander

**Multigrid methods for Helmholtz problems: A convergent scheme in 1D using standard components**

1. Introduction — 135
2. Smoothing — 139
   2.1 Smoothing analysis — 140
   2.2 Jacobi smoothing — 142
   2.3 Two-step Jacobi smoothing — 145
3. Coarse-grid correction — 152
   3.1 The Laplacian — 158
   3.2 The Helmholtz operator — 159
4. Two-grid iteration — 165
   4.1 The Laplacian — 165
   4.2 The Helmholtz operator — 167
5. Numerical examples — 175
   5.1 Two-grid experiments — 176
   5.2 Multigrid experiments, complexity — 178
6. Conclusions — 182

### Marcus J. Grote and Teodora Mitkova

**Explicit local time-stepping methods for time-dependent wave propagation**

1. Introduction — 187
2. Finite element discretizations for the wave equation — 190
   2.1 Continuous Galerkin formulation — 190
   2.2 Interior penalty discontinuous Galerkin formulation — 191
   2.3 Nodal discontinuous Galerkin formulation — 193
3. Leap-frog-based LTS methods — 194
   3.1 Second-order method for undamped waves — 195
   3.2 Fourth-order method for undamped waves — 199
   3.3 Second-order leap-frog/Crank–Nicolson-based method for damped waves — 202
4. Adams–Bashforth-based LTS methods for damped waves — 205
5. Numerical results — 211
   5.1 Stability — 211
   5.2 Convergence — 212
<table>
<thead>
<tr>
<th>5.3</th>
<th>Two-dimensional example</th>
<th>214</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Concluding remarks</td>
<td>215</td>
</tr>
</tbody>
</table>

Frédéric Nataf

**Absorbing boundary conditions and perfectly matched layers in wave propagation problems**

1 Introduction — 219
2 ABC — 220
2.1 Exact ABC — 221
2.2 Approximation of the exact ABC — 222
3 Plane waves analysis of an ABC — 224
4 Perfectly matched layers — 225
4.1 Helmholtz equation — 226
4.2 The wave equation — 228
5 Computation of the reflection coefficient of a PML — 229
6 Conclusion — 231

Roland W. E. Potthast

**Dynamic inverse scattering**

1 Introduction — 233
2 Reconstruction of time-dependent pulses by the point-source method — 236
3 Time-domain probe method (TDPM) — 238
4 Orthogonality sampling — 240
5 Dynamic inversion via data assimilation techniques — 240
5.1 Three-dimensional variational data assimilation — 242
5.2 Cycled probing and sampling method — 244
5.3 Partial reconstruction matching scheme — 245
6 Numerical examples — 247

Olaf Steinbach

**Boundary integral equations for Helmholtz boundary value and transmission problems**

1 Introduction — 253
2 Boundary integral equations — 255
2.1 Boundary integral operators — 255
2.2 Coercivity of boundary integral operators — 258
2.3 Injectivity of boundary integral operators — 260
2.4 Interior Robin boundary value problem — 264
2.5 Boundary integral equations for exterior boundary value problems — 266
3 Exterior Dirichlet boundary value problem — 266
3.1 Direct boundary integral equations — 267
3.2 Indirect boundary integral equations — 276
3.3 Regularised combined boundary integral equations — 279
4 Transmission problems — 280
4.1 Steklov–Poincaré operator equations — 281
4.2 Combined boundary integral equations — 285
5 Conclusions — 290

Color plates — 293

Index — 309